

Warm-up Exercises in Even and Odd

Math 130 *Kovitz*

1. Consider the function

$$f(x) = \frac{1}{(x+2)^2} + \frac{1}{(x-2)^2}.$$

(a) Method 1:

First see if you can simplify the expression to an equivalent form with a single fraction.

True or False?

- Since the domain is unbalanced, we can immediately conclude that this function must be neither even nor odd.
- Since the power of every x in the equivalent form is even, this function is immediately known to be even by the Rule of Even Powers.

(b) Method 2:

- Find $f(a)$ and $f(-a)$. How will they be related? What does that tell you about the function's being even or odd?
- Find $f(1)$ and $f(-1)$. What does this suggest about the function's being even or odd?

2. Consider the function

$$g(x) = \frac{1}{(x+2)^2} - \frac{1}{(x-2)^2}.$$

(a) Method 1:

True or False?

- Since the domain is unbalanced, we can immediately conclude that this function must be neither even nor odd.
- Since the powers in the given formula are all even, this function is immediately known to be even by the Rule of Even Powers.

(b) Method 2:

- Find $g(a)$ and $g(-a)$. How will they be related? What does that tell you about the function's being even or odd?
- Find $g(1)$ and $g(-1)$. What does this suggest about the function's being even or odd?

3. Consider the function

$$h(x) = \sqrt{x+10}.$$

ANSWERS to the PRACTICE for TEST 3

Is the function h even, odd, neither or both? There is a method to get an immediate answer.

Verify your answer by finding $h(6)$ and $h(-6)$.

4. In each part, try to decide which are possible with the given information: odd, even, neither, or both?

(a) $f(1) = 4$ and $f(-1) = 4$. Which two cases are excluded?

(b) $g(0) = 4$. Which two possibilities may be ruled out?

(c) $h(7) = 9$. Which one case may be ruled out?

(d) $f(-3) = 11$, $f(0) = 9$, and $f(3) = -11$.

What may be concluded (which one case)?

(e) $f(-2) = 2$, $f(2) = 2$, $f(-4) = 7$, and $f(4) = 8$.

What may be concluded (which one case)?

5. Define f so that for all $x \neq 0$, $f(x) = \frac{1-x^4}{x}$, and $f(0) = 5$.

Is f even, odd, neither, or both?

6. Define f so that for all $x \neq 0$, $f(x) = \frac{1-x^2}{x^4}$, and $f(0) = 10$.

Is f even, odd, neither, or both?

7. (a) Find a formula for an odd function f for which $f(0)$ is not defined.
(b) Find a formula for an even function f for which $f(0)$ is not defined.