Quadratic Function Problems

Math 130 *Kovitz*

1. Solve for x: $\quad 1 - 2x + \frac{3}{4}x^2 = 0.$

   Which of the following four methods did you decide was best?

   A) Clearing fractions and then factoring over the integers;
   B) Completing the square; C) The quadratic formula; D) Factoring fractions.

2. For each equation find

   - the vertex,
   - the line of symmetry, and
   - whether there is a maximum value or a minimum value.

   Then roughly graph the equation.

   (a) $y = (x + 14)^2 - 8$

   (b) $y = -9x^2$

   (c) $y = -2(x - 3)^2 + 6$

3. Complete the square, getting the given equation into the form $y = a(x - h)^2 + k$.

   (a) $y = x^2 + 14x + 37$

   (b) $y = 0.4x^2 + x + 1$

   (c) $y = -x^2 + 6x - 11$

   (d) $y = -2x^2 + 20x - 17$

4. Complete the square, getting the given equation into the form $y = a(x - h)^2 + k$.

   Then state the coordinates of the vertex and the intercepts of its graph.

   $$y = -2x^2 + 2x + 5/8$$

   (For practice, find the vertex by completing the square and also using the formula. Solve for the x-intercepts by each of the three commonly-used methods: completing the square, the quadratic formula, and factoring.)

5. Complete the square and graph the function, labeling the coordinates of the vertex, the x-intercepts (if any), and the y-intercept. Also state in which quadrant or quadrants the parabola lies.

   $$y = x^2 - 6x + 4$$

6. Write the quadratic function $f(x) = -\frac{1}{3}x^2 + 5x - 12$ in standard form and sketch its graph.

   Include the vertex, axis of symmetry, and all intercepts; and label them with both coordinates or the equation.

7. Show, by completing the square, that the graph of $y = 6x + x^2 + 18$ lies above the line $y = 7$.

8. A rock is tossed up from a raised platform, so that its position relative to ground level at t seconds is

$$y = f(t) = -5t^2 + 40t + 21.25 \text{ meters.}$$

   (a) Find the position of the rock relative to gound level at $t = 0$.

   (b) Sketch the graph of $f(t)$. Give coordinates of the highest point on the graph, and all the intercepts, even the one where $t < 0$.

   (c) What is the highest position of the rock?

9. For the rock in problem 8, compute and simplify $\dfrac{f(t+h) - f(t)}{h}$, the average velocity from time t to time $t + h$.

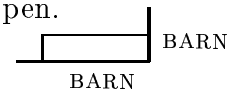
   Hint: check your answer by selecting specific values of t and h (such as $t = 3$ and $h = 1$) and computing the average velocity in two different ways: both from your simplified formula and from the positions of the projectile relative to the ground at times t and $t + h$ (here $t = 3$ and $t + h = 4$).

10. A number is multiplied by a second number, 16 smaller. What is the smallest possible product?

11. Find two numbers whose product is a maximum, under the condition that the sum of ten times the first number and six times the second number is 30.

12. Find the area of the largest rectangular field that 400 feet of fencing can enclose.

13. Find the largest area that a farmer can enclose by constructing a rectangular pen from 36 yards of fencing, if he uses a corner of his barn for two walls of the pen.



14. P is a function of H with the formula $P(H) = 50H - 10H^2$.

    Find the maximum value of P (the maximum value of the function).