Completing the Square Example

Math 130 *Kovitz* Spring 2012

By completing the square, put into standard form. Then check your answers by finding the vertex using the formula and showing that the result is the same.

This problem may be done as fractions or as decimals using a calculator. The fractional representation is preferred.

$$y = -4x^2 + 7x - 11$$

$y = -4\left(x^2 - \frac{7}{4}x \qquad\qquad \right) + 11$	Factor -4 out of the *first two* terms only, not out of the constant.
Make a box starting with $-\frac{7}{4}x$	Get (2nd line)$-\frac{7}{4}$, (3rd line) $-\frac{7}{4}/2 = -\frac{7}{8}$, (4th line) $\left(-\frac{7}{8}\right)^2 = \frac{49}{64}$
$y = -4\left[x^2 - \frac{7}{4}x + \frac{49}{64} - \frac{49}{64}\right] - 11$	Change to square brackets to empahsize the future distributive law. Add and subtract completing number *inside* the parentheses.
$y = -4\left[\left(x - \frac{7}{8}\right)^2 - \frac{49}{64}\right] - 11$	The first three terms inside the box constitute the perfect square. Replace the first three terms with the perfect square that is found from the box. Combine the letter x obtained from the first line of the box with the number $-\frac{7}{8}$ obtained from the third line of the box. Enclose this binomial in parentheses, then square.
$y = -4\left(x - \frac{7}{8}\right)^2 - 4\left(-\frac{49}{64}\right) - 11$	Use the distibutive law of multiplication over subraction to get rid of the square parentheses. Don't forget to **distribute the dangling term.**
$y = -4\left(x - \frac{7}{8}\right)^2 + \frac{49}{16} - 11$	Multiply, then simplify the fraction.
$y = -4\left(x - \frac{7}{8}\right)^2 + \frac{49}{16} - \frac{176}{16}$	Get common denominator. Next step will be combining the fractions to get the final answer.

$$y = -4\left(x - \frac{7}{8}\right)^2 - \frac{127}{16}$$

Check:

The vertex formula gives $\left(-\frac{b}{2a},\ c - \frac{b^2}{4a}\right) = \left(\frac{-7}{-8},\ -11 - \frac{49}{-16}\right) = \left(\frac{7}{8},\ -\frac{127}{16}\right)$.

From our answer, the vertex is $\left(\frac{7}{8},\ -\frac{127}{16}\right)$. It checks.

Decimals:

This is much messier because you need to keep the exact decimal with several places.

The key lines become: $y = -4(x^2 - 1.75x \qquad\qquad) - 11$.

Box is: $-1.75x$, -1.75, -0.875, $(-0.875)^2 = 0.765625$.

Then: $y = -4[x^2 - 1.75x + .765625 - .765625] - 11$, $y = -4[(x - 0.875)^2 - .765625] - 11$, $y = -4(x - 0.875)^2 - 4(-.765625) - 11$,
$y = -49x - 0.875)^2 + 3.0625 - 11$, and $y = -4(x - 0.875)^2 - 7.9375$.

The vertex formula will be $\left(\frac{-7}{-8},\ -11 - \frac{49}{-16}\right) = (0.875,\ -11 + 3.0625) = (0.875,\ -7.9375)$. So, it checks.