Summary of the Key Steps in Completing the Square:

1. Isolate the constant term.
2. Factor out the a by dividing the term bx by a to get $\frac{b}{a}x$. Do not factor out of the constant.
3. Replace the parentheses by square brackets.
4. Determine the completing number. Divide the coefficient of x by 2; this will give the second term of the binomial that was squared. Square that second term; this is the completing number.
5. Add and subtract the completing number *inside* the square brackets.
6. Replace the first three terms inside with the square of the binomial.

   Combine the variable letter with the unsquared constant from the completing process above. This goes in parentheses and is squared.
7. Distribute to the squared binomial and to the dangling term.
8. Combine the constant found by multiplying the dangling term by a with the original constant c.

Example

$$-\frac{1}{3}x^2 - 3x + 4$$

$-\frac{1}{3}x^2 - 3x \qquad + 4$

$-\frac{1}{3}(x^2 + 9x) + 4$

$-\frac{1}{3}[x^2 + 9x] + 4$

$9x$ becomes 9 becomes $9/2$ becomes $81/4$.

$-\frac{1}{3}[x^2 + 9x + 81/4 - 81/4] + 4$

$-\frac{1}{3}[(x + 9/2)^2 - 81/4] + 4$

$-\frac{1}{3}(x + 9/2)^2 - \frac{1}{3}\left(-\frac{81}{4}\right) + 4$

$-\frac{1}{3}(x + 9/2)^2 + \frac{27}{4} + 4$

$-\frac{1}{3}(x + 9/2)^2 + \frac{43}{4}$

The vertex will be at $(-9/2, 43/4)$.

Vertex formula: $a = -\frac{1}{3}$, $b = -3$, $c = 4$. Result $h = -b/2a = -(-3)/(-\frac{2}{3}) = -4.5$ and

$$k = c - b^2/4a = 4 - [(-3)^2/(4(-\tfrac{1}{3})] = 4 + 9/(\tfrac{4}{3}) = 4 + \tfrac{27}{4} = \tfrac{43}{4}.$$

Try also:

$x^2 + x + 1$

$7x^2 + 4x + 1$

$2x^2 + 3x + 1$

$4x^2 - \frac{1}{2}x + 1$