Parabola Graphing Examples

Math 130 *Kovitz*

1. Complete the square of

$$y = -\frac{1}{5}x^2 + 5x - 20,$$

getting the equation into the form

$$y = a(x - h)^2 + k.$$

What are the values of h and k?

Check by applying the formulas for h and k in terms of a, b, and c.

2. Consider the quadratic function

$$y = 2x^2 + 2x + 1.$$

- Complete the square.
- State—as an ordered pair—the coordinates of the vertex of the graph.
- Write the equation of the line of symmetry.
- Solve the equation $0 = 2x^2 + 2x + 1$.
- State—as ordered pairs—the coordinates of the y-intercept of the graph and of its symmetric partner.
- State—as ordered pairs—the coordinates of the x-intercepts (if any) of the graph.
- Plot, with coordinates, one point in each quadrant where you have not yet plotted a point.
- **Graph the equation,** using the points you found above.

3. Consider the quadratic function

$$y = 2x^2 + 5x + 3.$$

- Complete the square.
- State—as an ordered pair—the coordinates of the vertex of the graph.
- Write the equation of the line of symmetry.
- Solve the equation $0 = 2x^2 + 5x + 3$.
- State—as ordered pairs—the coordinates of the y-intercept of the graph and of its symmetric partner.
- State—as ordered pairs—the coordinates of the x-intercepts (if any) of the graph.
- Plot, with coordinates, one point in each quadrant where you have not yet plotted a point.
- **Graph the equation,** using the points you found above.

Answers follow.

Answers.

1. $y = -\frac{1}{5}x^2 + 5x - 20.$

   $y = -\frac{1}{5}(x^2 - 25x) - 20.$

   $y = -\frac{1}{5}\left[x^2 - 25x + 625/4 - 625/4\right] - 20.$

   $y = -\frac{1}{5}\left[(x - 25/2)^2 - 625/4\right] - 20.$

   $y = -\frac{1}{5}(x - 25/2)^2 - \frac{1}{5}(-625/4) - 20.$

   $y = -\frac{1}{5}(x - 25/2)^2 + 125/4 - 20.$

   $y = -\frac{1}{5}(x - 25/2)^2 + 45/4.$

   The values of (h, k) are $(12.5, 11.25)$.

   Start with $a = -1/5$, $b = 5$, and $c = -120$.

   It gives
   $h = (-5)/(-2/5) = 25/2$ and
   $k = c - b^2/4a = -20 - 25/(-4/5) = -20 + 125/4 = 45/4.$

2. Consider the quadratic function

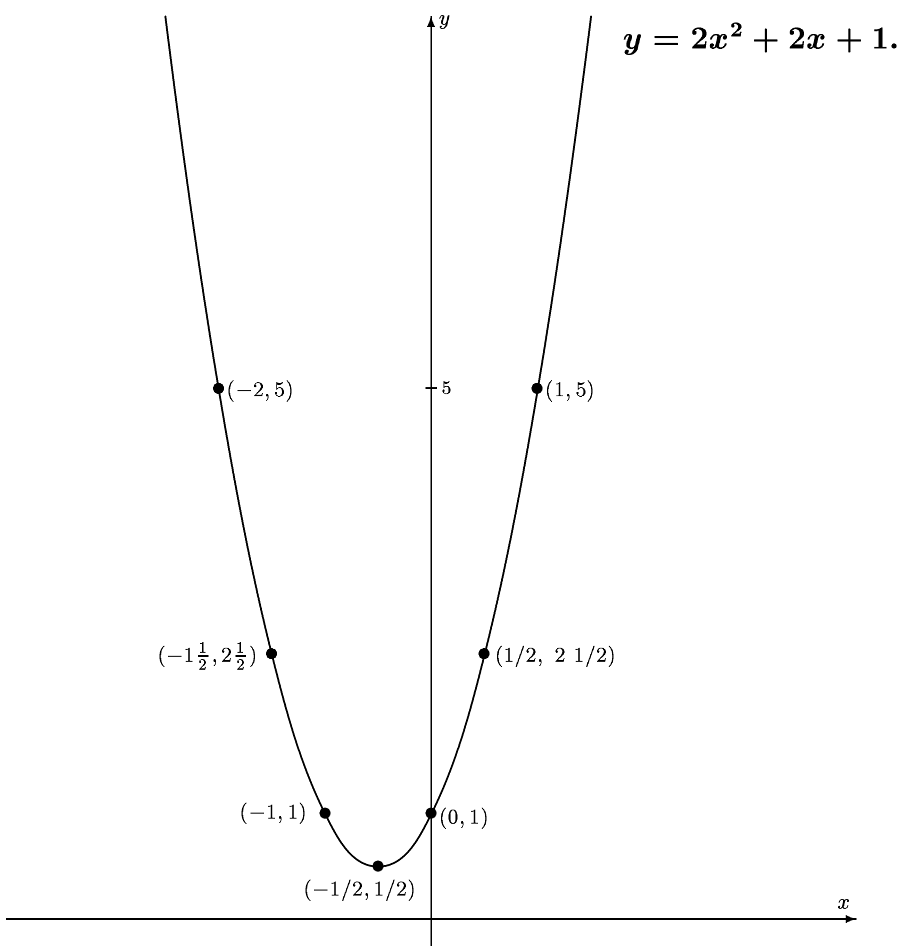
$$y = 2x^2 + 2x + 1.$$

- $y = 2(x^2+x)+1 = 2[x^2+x+1/4-1/4]+1 = 2(x+1/2)^2-1/2+1 = 2(x+1/2)^2+1/2.$

  Key Steps in Completing the Square:

  (a) Factor out the 2 by dividing the term $2x$ by 2 to get x.
  (b) Determine the completing number. From x, get 1, then 1/2, then $(1/2)^2 = 1/4$.
  (c) Replace the parentheses with square brackets.
  (d) Add and subtract 1/4 *inside* the square brackets.
  (e) Replace the first three terms inside with the perfect square $(x + 1/2)^2$.
  Combine the initial letter with the third step from the completing process above. So (x, then 1/2) will give $(x + 1/2)$.
  (f) Distribute to the dangling term. So $2(-1/4)$ becomes $-1/2$.
  (g) Combine the two constants: $-1/2 + 1 = 1/2$.

- $(-0.5, 0.5)$.
- $x = -0.5$.
- $x = \dfrac{-2 \pm \sqrt{4-4(2)(1)}}{4} = \dfrac{-2 \pm \sqrt{-4}}{4}$, so there are no real solutions.
- $(0,1)$ and $(-1,1)$.
- There are no x-intercepts.
- The misssing quadrant is the first quadrant and two points are: $(1/2, 2\frac{1}{2})$ and $(1,5)$.



3. Consider the quadratic function

$$y = 2x^2 + 5x + 3.$$

- $y = 2(x^2 + \frac{5}{2}x) + 3 = 2[x^2 + \frac{5}{2}x + 25/16 - 25/16] + 3 = 2(x + 5/4)^2 - 2(25/16) + 3 = 2(x + 5/4)^2 - 25/8 + 3 = 2(x + 5/4)^2 - 1/8.$

  Key Steps. Factor out $5x/2$; Get completing no.: $\frac{5}{2}x$ to $5/2$ to $5/4$ to $(5/4)^2 = 25/16$; Add and subtract $25/16$ inside the square brackets; Create the perfect square from x and $5/4$, and replace first three terms: $(x + 5/4)^2$; Distribute to the dangling term: $2(-25/16) = -25/8$; Combine the constants: $-25/8 + 3 = -25/8 + 24/8 = -1/8$.

- $(-1.25,\ -0.125)$.
- $x = -1\,1/4$ or $x = -1.25$.
- $x = \dfrac{-5 \pm \sqrt{25 - 4(2)(3)}}{4} = \dfrac{-5 \pm \sqrt{1}}{4}$. $x = -1$ or $x = -1.5$.
- $(0, 3)$ and $(-2.5, 3)$.
- $(-1.5, 0)$ and $(-1, 0)$.
- The misssing quadrant is the first quadrant and two points are: $(1/2,\ 6)$ and $(1, 10)$.