Composition and Inverse Problems

Math 130 *Kovitz*

1. Let $f(x) = x^2 + 3$ and let $g(x) = x - 2$.
   (a) Find $f \circ g$ and $g \circ f$.
   (b) Find $(f \circ g)(5)$ and $(g \circ f)(5)$.

In Problems 2–6, let f be the function given by

$$f(x) = 9x - 36.$$

2. Find the graph of the inverse of f by three methods:
   (a) by reflecting the graph of f across the diagonal line $y = x$,
   (b) by reflecting points on the graph of f and connecting them,
   (c) by producing an equation for the inverse function and graphing it.
3. Find a formula for $f^{-1}(x)$.
4. Find $f^{-1}(f(9))$ and $f(f^{-1}(9))$.
5. Test f for symmetry across the line $y = x$.
6. Compare $f^{-1}(9)$ and $[f(9)]^{-1}$. Are they equal?

7. Find a formula for the inverse function.
   (a) $f(x) = 2/x$.
   (b) $g(x) = \dfrac{8x + 3}{5}$.
   (c) $h(x) = \sqrt{1 + \dfrac{1}{x}}$.
   (d) $i(x) = \dfrac{2x + 7}{3x - 5}$.
   (e) $j(x) = \dfrac{1}{x^3 - 7}$. (Why do the answers derived symbolically and verbally differ?)
   (f) $k(x) = -x$.
   (g) $f(x) = -x + 1$.

8. Test f for symmetry across the line $y = x$.
   (a) $y = (x + 2)^2$
   (b) $y = 15 - x$
   (c) $y + 7 = \dfrac{1}{x + 7}$
   (d) $y = \dfrac{9 - 2x}{x + 2}$
   (e) $y = \sqrt{x}$
   (f) $x^2 - xy + y^2 = 11$ Find all of the four usual symmetries that apply. Don't assume this equation represents a function.

In Problems 9–12, let f be the function given by

$$f(x) = \frac{109 - 5x}{5 + 3x}.$$

9. Write an equation of the inverse relation.

10. Test the graph of $y = \frac{109-5x}{5+3x}$ for symmetry across the line $y = x$ (that is, determine whether $y = \frac{109-5x}{5+3x}$ is its own inverse).

    What is y when $x = 1$, 2, 4.2, 5, 9, 13?

    Do those results agree with your conclusion about symmetry across the line $y = x$?

11. Is the inverse relation a function?

    (If it is a fuction) Call the function f.

    Revisit problem 10 by finding $f(f(1))$, $f(f(2))$, $f(f(4.2))$, $f(f(5))$, $f(f(9))$, and $f(f(13))$.

    If this trend continued for $f(f(a))$ for all a, what would it tell us about symmetry of f across the line $y = x$?

12. If you answered yes to Problem 11, find a formula for $f^{-1}(x)$, and recheck your solution to Problem 10 in light of that formula.

13. Write the function as a composite of two functions f and g. (More than one answer is correct.)

    (a) $(f \circ g)(x) = f(g(x)) = 2(3x - 1)^2 + 5(3x - 1)$

    (b) $(f \circ g)(x) = f(g(x)) = \dfrac{2}{\sqrt{4 - x^2}}$

    (c) $(f \circ g)(x) = f(g(x)) = \dfrac{6 - x}{1 - x}$

14. Rewrite $h(x) = \frac{6-x}{1-x}$ as a verbal composition string, where each operation is as simple as possible. *This is not easy.*

    From this, find a verbal composition string for the inverse of h.

    Starting with an arbitrary number a, show that $\frac{6-x}{1-x}$ and its inverse are the same function. (This result will establish that the inverse relation is a function and that the original relation is symmetric across the line $y = x$.)