The Exponent Function

Math 130 *Kovitz*

Definition of exponent and exponential equations

An exponent equation is an equation like $y = 2^x$, with the exponent the independent variable x. There's a unique output upon substitution, so it's a function. Don't confuse it with the quadratic function $y = x^2$, or generally with the power function $y = x^c$ with c a real number, where the input variable is in the base of the expression.

An equation, such as $y = 2^{x+1}$, where the exponent is a more complcated variable expression, or an equation of the form $y = c \cdot a^x$, such as $y = 3 \cdot 2^2 = x$, will be called an exponential equation.

Definition of an Exponent Function

An exponent function is a function of the form $f(x) = a^x$, where a is a positive number other than 1 and x is any real number. The base is a and the exponent is x.

We shall also call that function "the exponent function base a" and write it as $f(x) = \exp_a x$.

The name of the function is $\exp_a$ and its formula is $y = a^x$.

Such a function is not an algebraic function. It is a transcendental function. The function $\exp_a x = a^x$ should be called the exponentiation function since the exponent is the *input* and the output is the exponentiated base (the base raised to the power). For example look at $\exp_2 5$. The exponent, 5, is the input and the output is 2^5, the 5th power of 2, the base raised to the input. This will take on greater importance when we look at the inverse of an exponent function.

Some Properties of Exponent Functions

- The domain consists of all real numbers. $(-\infty, \infty)$
- The range is the positive real numbers. $(0, \infty)$
- The graph will have points in the first and second quadrants.
- The y-intercept is $(0, 1)$.
- There is no x-intercept.
- The graph is continuous because it is everywhere a connected curve.
- If $a > 1$: The function is increasing and the graph rises from left to right. Also the negative x-axis is a horizontal asymptote.
- If $0 < a < 1$: The function is decreasing and the graph falls from left to right. Also the positive x-axis is a horizontal asymptote.
- Any exponent function has the same shape as $y = 2^x$. Depending on the value of a, the base, it could be stretched or shrunk in the x-direction (when $a > 2$ it's shrunk making the graph narrower, and when $1 < a < 2$ it's stretched, making the graph more flat-shaped). If the base is less than 1, we know that the base $a = 1/c$, where $c > 1$. So, in addition the graph is a reflection of the graph of $y = c^x$ across the y-axis. That is because $a^x = \left(\frac{1}{c}\right)^x = c^{-x}$.
- The graph is concave up.
- The function is a one-to-one function since its graph passes the horizontal-line test and, as a result, has an inverse function.
- There is no maximum or minimum value of y, and the graph has no endpoint.
- The graph of an exponent function of the form $y = a^x$ has no symmetries. This will not necessarily be true for the graph of an exponential function. (An exponential function is either an exponent function multiplied by a constant or a function with a fixed base and a more complex exponent.)