Logarithm and Exponential Review

Math 130 *Kovitz*

Definition $\log_a x =$ *the power to which you raise* a *to get* x. (*Base* $a > 0$)

Rules.

$$\left.\begin{aligned} a^{x+y} &= a^x \cdot a^y \\ \log_a x + \log_a y &= \log_a(x \cdot y) \end{aligned}\right\} \text{ convert between addition and multiplication}$$

$$\left.\begin{aligned} a^{x \cdot y} &= (a^x)^y \\ y \log_a x &= \log_a x^y \end{aligned}\right\} \text{ subsequent powers}$$

$$\left.\begin{aligned} a^x &= b^{x \log_b a} \\ \log_a x &= \log_b x / \log_b a \end{aligned}\right\} \text{ change of base}$$

Exercises

True or False?

1. $\log_a(x+y) = \log_a x + \log_a y$
2. $\exp_a(x+y) = (\exp_a x)(\exp_a y)$
3. $\log_a\left(\frac{x}{y}\right) = \dfrac{\log_a x}{\log_a y}$
4. $\exp_a\left(\frac{x}{y}\right) = \sqrt[y]{a^x}$
5. $\dfrac{\log_a x}{\log_a y} = \log_a x - \log_a y$
6. $\dfrac{\exp_a x}{\exp_a y} = \exp_a(x-y)$
7. $x \log_a y - 1 = \log_a(y^x/a)$
8. $\exp_a(xy) = \exp_a x + \exp_a y$

Answers.

1. F 5. F
2. T 6. T
3. F 7. T
4. T 8. F

Find the decimal value of x.

9. $4^{2\log_4 x} = 9$
10. $3^{2x+4} = 9^{5x}$
11. $3^x = \frac{1}{2}$
12. $3^{x+2} - 4 = 3^x$
13. $2^x = 3^{x-1}$
14. $4^x = 2 \cdot 2^x + 8$
15. $\log_3(2x+1) = \log_3\left(\frac{1}{x}\right) + 1$
16. $(\log_3 x)^2 + \log_3(\frac{1}{x^2}) = 8$
17. $\log_4(x+3) = \log_2(x-3)$
18. $\log_9(3-2x) = \log_3 x$
19. $\log_2 x = 1 + \log_{10} x$
20. $\log_x 8 = 3/2$
21. $\log_4(x+3) + \log_4(x-3) = 2$
22. $\log x = 1 - \log(x-3)$
23. $\log_{10} \log_3 x = 1$
24. $\log_3 \log_{10} x = 1$
25. $\log_3 \log_5 x = -1$
26. $(\log_2 x)^2 = 5 + 4\log_2 x$
27. $27 = 8^{\log_2 x}$
28. $x^{\log x} = 10^5 x^4$

Compute as a decimal.

29. $\log_4 8$
30. $\log_8 4$
31. $\log_{\frac{1}{2}} 6$
32. $\log_5 5$

Simplify.

33. $9^{\log_3 2}$
34. $3^{\log_9 2}$
35. $\log_2(8^{x+1})$
36. $16^{(-1+\log_4 3)}$

Find the value as a fraction.

37. $9^{\log_3(2)-1}$
38. $8^{1-\log_2(3)}$

Write in the form Ca^x

39. $10^{1.4771213-x}$
40. $10^{2x-.30103}$
41. Find $\log_2 A$ given that $\log_8 A = a$.
42. Find $\log_{1/b} A$ given that $\log_b A = 3$.

Graph the function. Find the asymptote and the decimal value of all intercepts, and plot four points.

43. $f(x) = \exp_3(x-1) - 4$
44. $g(x) = \exp_2(x+2) - 5$
45. $h(x) = \log_3(x+1) + 1$
46. $k(x) = \log_2(x+2) - 3$

Answers.

9. 3 10. 1/2 11. $-.6309298$
12. $-.6309298$ 13. 2.71 14. 2
15. 1 16. 81, 1/9 17. 6
18. 1 19. 2.695731 20. 4
21. 5 22. 5 23. 3^{10}
24. 10^3 25. $\sqrt[3]{5}$ 26. 32, 1/2
27. 3 28. 10^5, 1/10
29. 3/2 30. 2/3 31. -2.5849625
32. 1 33. 4 34. $\sqrt{2}$
35. $3x+3$ 36. 9/16 37. 4/9
38. 8/27 39. $\approx 30(1/10)^x$
40. $\approx \frac{1}{2}100^x$ 41. $3a$ 42. -3
43. (x-int) 2.2618595
44. (x-int) 0.321928
45. (x-int) $-2/3$ 46. (x-int) 6