Simplified Exponential Growth and Decay Formuls

Math 130 *Kovitz*

An exponential function of time can be written in the form

$$y = f(t) = C_0 a^t.$$

Consider the input and output. The input is a value of time, the variable t. The output is a value of the variable, y, which is $f(t)$.

A completely verbal way of looking at this is to state:

> Every time one adds h to the input, the output is multiplied by a^h, a being the base of the function.
>
> In this setup, h is simply Δt, the change in t. Since t represents time, Δt is the elapsed time.

It is useful to apply it in this way:

> The base raised to the elapsed time is equal to the ratio of the outputs.

As a formula:

$$a^{\Delta t} = y_2/y_1.$$

Example:

Let $C_0 = 3$, and $a = 2$.

So $f(t) = 3 \times 2^t$.

Start with $f(-1) = 3/2 = 1.5$.

Now add 4 to the input. This makes the new input 3.

The original output will be multiplied by $2^4 = 16$, so the new output must be $1.5 \times 16 = 24$. Check it in the original formula: $3 \times 2^3 = 3 \times 8 = 24$.

The second verbal method will tell us: $2^4 = y_2/y_1$. And indeed, $16 = 24/1.5$.

How is this useful? Assume the values of the base, x_1, x_2, and the initial output. From them, find the new output, called x.

$2^{(3-(-1))} = x/1.5$. Solving it leads to $x = 24$.