Expo Growth Problems
Math 130 *Kovitz*

1. Wasilla, Alaska, has a population that is growing exponentially. At the start of the year 1990 it was 4028 while at the start of the year 2010 it was 7831.

   Find (assuming a constant rate of growth)

   (a) the one-year growth factor.

   (b) the annual percentage gain.

   (c) the doubling time.

   (d) the projected population at the start of the year 2100.

2. A club has an exponentially-growing membership. Four years ago there were 837 members, but now there are 11,876 members.

   Find (assuming a constant rate of growth)

   (a) the one-year growth factor.

   (b) the annual percentage gain.

   (c) the doubling time.

   (d) the projected membership twenty-four years from now.

   (e) Practically, what is wrong with this model?

3. The actual world population in mid-year 2050 was about 2.5560 billion. During the period 1950-2000, world population increased at a rate the doubled the population approximately every 40 years.

   (a) Find a formula that estimates the mid-year world population for 1950-2000, assuming that the rate of growth was fairly constant during that period.

   (b) In mid-year 1960, the actual world population was 3.0394 billion.
   Using the formula from part (a), estimate the world population in mid-year 1960.

Answers follow.

Answers.

1. (a) $\sqrt[20]{\dfrac{7831}{4028}} = \sqrt[20]{1.944141013} = 1.033799667.$

   (b) 3.38%.

   (c) 20.852 years, or approximately 20 years and 311 days.

   (d) 155,998.

2. (a) $\sqrt[4]{\dfrac{11876}{837}} = \sqrt[4]{14.18876941} = 1.940824128.$

   (b) 94.08%.

   (c) 1.0453 years, or approximately one year and sixteen-and-a-half days.

   (d) 96,903,301,658—which is about 97 billion.

   (e) This result tells us that the constant rate of exponential growth must surely come to an end before those twenty-four years are up.

3. (a) Using the formula and data above, we see that the mid-year world population in the year t, expressed in billions, was approximately

   $$2.556 \cdot \left(2^{1/40}\right)^{t-1950} = 2.556 \cdot 2^{(t-1950)/40}.$$

   The value $2^{1/40} = \sqrt[40]{2} \approx 1.017479692$ is the base of the exponential function. That means each year the estimate of the world population increased by about 1.7479692% over the previous year.

   (b) 3.0396 billion.