

Exponential Growth Example

Math 130 *Kovitz*

The size of a bacteria colony grows exponentially. Initially there were 612 bacteria; but after 8 hours there were 1823 bacteria present. Assume the rate of growth is constant.

1. Write in general form an expression which will give the size of the colony as a function of t in hours; then rewrite that function, replacing all constants with numbers that apply to the given data.
2. Find the hourly percentage increase in the number of bacteria present.
3. Find the number of bacteria present 80 hours after the initial time.
4. Find the doubling time. Based on that, when will the number of bacteria be 4896?
5. When will there be one million bacteria present?

Answers follow.

Answers.

1. The unknown base raised to the elapsed time equals the output ratio.

$$\text{So: } a^8 = 1823/612 = 2.97875817 \text{ and } a = \sqrt[8]{\frac{1823}{612}} = 1.146184169.$$

Then the equation is $f(t) = C_0 a^t$, and for this example

$$f(t) = 612(1.146184169)^t.$$

2. The hourly percentage is one hundred percent times (the base minus 1).

$$\text{Here: } (1.14618 - 1) \times 100\% = 14.618\%.$$

3. $f(80) = 612(1.146184169)^{80} = 612(54998.70515) = 33,658,208.$

4. $D = \log 2 / \log a = 0.301029995 / 0.059254405 = 5.080297263$ hours.

The number 4896 is eight times as much as the initial value, so three doublings would have occurred. Three times the doubling time is about 15.24 hours.

5. $1,000,000 = 612(1.146184169)^t.$

$$1633.986928 = (1.146184169)^t, \text{ after dividing both sides by 612.}$$

$$\log 1633.986928 = \log[(1.146184169)^t]. \text{ The unknown is in the exponent; solve by taking common logs.}$$

$$\log 1633.986928 = t[\log(1.146184169)]. \text{ The power within a log comes out as a multiplier in front.}$$

$$t = \frac{\log 1633.986928}{\log 1.146184169} = \frac{3.213248578}{0.059254405} = 54.228 \text{ hours. To solve for } t, \text{ just divide by its coefficient.}$$

This means that 54.228 hours after the initial time (a few hours more than two days) there will be one million bacteria present in that colony, if we assume exponential growth at a constant rate.