Doubling Time Examples

Math 130, Kovitz

Once a is known, it is in some cases possible to estimate the doubling time without using a calculator.

Use the formula

$$D = \log 2 / \log a$$

and approximate $\log 2$ with the value 0.3 (except where it is indicated to use the more exact value: 0.301).

A. Taking the square root of a has what effect on the doubling time? Why?

B. Squaring a has what effect on the doubling time? Why?

Example: For $a = 5$; we get $\log 2 / \log 5 \approx 3/7$.

For $a = \sqrt{5}$; we get $\log 2 / \log \sqrt{5} \approx 6/7$, twice as much as for $a = 5$.

For $a = 25$; we get $\log 2 / \log 25 \approx 3/14$, half as much as for $a = 5$.

1. Estimate the doubling time when $a = 5/4$.

   From this estimate, get an immediate estimate of the doubling time when $a = 25/16$.

   From this estimate, get an immediate estimate of the doubling time when $a = \sqrt{5}/2$.

2. Using the more exact value of $\log 2 = 0.301$, estimate without a calculator the doubling time when $a = \frac{8}{25}\sqrt{10}$.

   How is $128/125$ related to $\frac{8}{25}\sqrt{10}$?

   Use that fact and the earlier result to get an immediate estimate of the doubling time for $a = 128/125$.

   How is $\frac{2}{5}\sqrt[4]{40}$ related to $\frac{8}{25}\sqrt{10}$?

   Use that fact and the result when $a = \frac{8}{25}\sqrt{10}$ to get an immediate estimate of the doubling time for $a = \frac{2}{5}\sqrt[4]{40}$.

Answers follow.

A. It doubles, because $\log \sqrt{a} = \frac{1}{2} \log a$.

B. It becomes half as much, because $\log(a^2) = 2 \log a$.

1. 3

   1.5

   6

2. 60

   It is the square of $\frac{8}{25}\sqrt{10}$.

   30

   It is the square root of $\frac{8}{25}\sqrt{10}$.

   120