Doubling Time Estimation

Math 130, Kovitz

For each value of a, estimate the doubling time without using a calculator.

Use the formula

$$D = \log 2/\log a$$

and approximate $\log 2$ with the value 0.3 (except where it is indicated to use the more exact value: 0.301). For $\ln 2$, use the approximation 0.693.

Examples: For 2; we get $\log 2/\log 2 = 1$. For $\sqrt{2}$; we get $\log 2/\log\sqrt{2} = 2$.
For $\sqrt[3]{2}$; we get $\log 2/\log\sqrt[3]{2} = 3$. For 4; we get $\log 2/\log 4 = 1/2$.

It should be observed as the values proposed for a (listed below) increase, the doubling time will decrease.

1. 128/125=1.024 (use $\log 2 \approx 0.301$)
2. $\frac{4}{5}\sqrt{2}$
3. $\sqrt[4]{2}$
4. 5/4=1.25
5. 32/25=1.28
6. $\frac{128}{125}\sqrt{2}$ (use $\log 2 \approx 0.301$)
7. 8/5=1.60
8. $\frac{5}{4}\sqrt{2}$
9. $\frac{32}{25}\sqrt{2}$
10. $\frac{8}{5}\sqrt{2}$
11. 2.5
12. e
13. $\sqrt{8}$
14. $2.5\sqrt{2}$
15. 5
16. $\sqrt{32}$
17. $5\sqrt[3]{2} \approx 6.300$
18. $5\sqrt{2} \approx 7.07$
19. 10
20. $\sqrt{128}$
21. 1000

Which of these have doubling times that are exact rational numbers?

Answers follow.

Note that these answers are decreasing (some are equal, because they are only approximations).

1. Around 30. This is the only estimate that is a bit off, even as a rough approximation.
2. 6
3. 4
4. 3
5. 3
6. $1\frac{7}{8}$
7. 1.5
8. 1.2
9. 1.2
10. 6/7
11. 0.75
12. $\ln 2 \approx 0.693$
13. 2/3
14. 6/11
15. 3/7
16. 2/5
17. 3/8
18. 6/17
19. 0.301
20. 2/7
21. 0.1

Only $\sqrt[4]{2}$, $\sqrt{8}$, $\sqrt{32}$, and $\sqrt{128}$ had exact rational solutions.