A Tale of Exponential Growth and Decay
Math 130 *Kovitz*

Harry and Ralph were classmates in a prominent New England business school. Their lifetime friendship remained strong despite geographic separation and the burdens of career, marriage, and family. One aspect of the relationship between the two men is a good-natured competitiveness.

The two graduates were fortunate in that each had in 1976 a sum of $45,000 to invest. Harry, always the conservative thinker, decided to put his money into something solid, Boston real estate. He purchased a triple-decker house in Dorchester. Ralph was speculative and looked for an investment that while risky offered the prospect of a substantial gain. In addition he espoused the philosophy that in the markets the successful investor is the person who goes against the commonly-held notions of the crowd. He saw that almost everybody felt that Bolivian currency was destined to be worthless in the future as a result of continuing rampant inflation. To carry his ideas to the extreme, he decided to invest in Bolivian currency, speculating that the masses and the experts were all wrong and that he would get rich.

Harry's investment proved to be as solid as he predicted. It increased exponentially with an annual increase of 8%.

Ralph's investment did not do very well. It decreased exponentially with an annual decrease of 40% because of the economic instability in Bolivia.

How much was Harry's house worth in 1999?

How much was Ralph's stockpile of bolivianos worth in 1999?

At the twentieth reunion of the business school the two friends got together. Harry ribbed Ralph about his investment.

"I see you never learn. Tell me your next idea so I can avoid it."

"Forget about my troubles. Let's talk about your investment.
"I was looking at a list of the value of such a house in Boston for each of the last twenty years. It is really exciting that the value moves up so unpredictably. You must be very excited each year."

"What are you talking about! What unusual appreciation! That investment is the most boring thing I ever followed. Every year the logarithm of the investment goes up by exactly the same amount."

How much did the logarithm of the value of Harry's house increase each year?

"So, Ralph, what about the value of your investment. It really took a nosedive."

"Not really. Just like yours, the logarithm of my investment changed exactly the same amount every year. Unfortunately, it was a decrease."

How much did the logarithm of Ralph's investment decrease every year?

"Harry, you idiot! Why are you making such a big deal about your real estate? The logarithm of its value only increases 0.033423755 every year, and even in the 23 years since you bought that stupid house the logarithm of its value has only increased 0.768746376 in total."

What should Harry reply to preserve his reputation as a sharpie?