Angles, Standard Position on the Unit Circle, and Trig Ratios

Math 130 *Kovitz*

Measuring an angle

Assume that the vertex of an angle is at the center of any circle. The size of a central angle is proportional to the length of the opposite arc.

One revolution (1 rev) is the angle subtended by a circumference of the circle.

Define a degree (1°) as 1/360th of 1 rev; a minute ($1'$) as 1/60 of one degree; a second ($1''$) as 1/60 of one minute.

One grad is defined as 1/400 th of one revolution. We don't work with this measure; it is used in engineering.

The radian measure of an angle, θ, is defined as the ratio of the length of the opposite arc divided by the radius of the circle.

$$\theta = \frac{s}{r}.$$

The circumference of a circle

$C = \pi \times d = 2\pi \times r.$

Conversions between radian and degree measure

Use the fact that $180^\circ = \pi$ radians; then proportionalize. One radian is then approximately 57.29577951°.

Formula for the length of the opposite arc

$$s = r \times \theta.$$

Also the radius may be found by using $r = \frac{s}{\theta}$.

These formulas are only valid when θ is expressed in radian measure.

Angular Velocity

Angular velocity: $\omega = \Delta\theta/\Delta t$. Linear velocity along the rim: $\nu = \Delta s/\Delta t = r\omega$.

Triangles

Any triangle: has 3 sides and 3 interior angles, the interior angles add to 180° or π radians, a larger side is opposite a larger angle, and the sum of the lengths of any two sides is greater than the length of the third side

Any right triangle: has a right angle of 90 degrees or $\pi/2$ radians, the side opposite is called the hypotenuse, the sum of the two acute angles is 90 degrees or $\pi/2$ radians, and the sum of the squares of the two legs equals the square of the hypotenuse ($a^2 + b^2 = c^2$).

Trigonometric ratios in a right triangle (memorize by: SOH, CAH, TOA)

$\sin\theta = \frac{\text{side opposite } \theta}{\text{hypotenuse}}$ $\csc\theta = \frac{\text{hypotenuse}}{\text{side opposite } \theta}$

$\cos\theta = \frac{\text{side adjacent to } \theta}{\text{hypotenuse}}$ $\sec\theta = \frac{\text{hypotenuse}}{\text{side adjacent to } \theta}$

$\tan\theta = \frac{\text{side opposite } \theta}{\text{side adjacent to } \theta}$ $\cot\theta = \frac{\text{side adjacent to } \theta}{\text{side opposite } \theta}$

These ratios depend only on the size of the angle and not on the particular triangle.

It can be shown that:

for $\theta = \pi/6$: $\sin = 1/2$ and $\cos = \sqrt{3}/2$, for $\theta = \pi/4$: $\sin = \cos = \sqrt{2}/2$, and for $\theta = \pi/3$: $\sin = \sqrt{3}/2$ and $\cos = 1/2$.

Useful identities are $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\cos(90^\circ - \theta) = \sin\theta$, and $\sin(90^\circ - \theta) = \cos\theta$.

The last two can be expressed in radians: $\cos(\pi/2 - \theta) = \sin\theta$, and $\sin(\pi/2 - \theta) = \cos\theta$.

Also: $\sin^2\theta + \cos^2\theta = 1$, $1 + \tan^2\theta = \sec^2\theta$, and $1 + \cot^2\theta = \csc^2\theta$.

Standard position on the unit circle

Any real number will correspond to the terminal point of an arc on the unit circle starting at $(1, 0)$, the rightmost point of the circle, having length equal to the real number, and being oriented in the appropriate direction (counterclockwise for positive reals and clockwise for negative reals).

Any point on the unit circle is represented by many different real numbers that differ by multiples of 2π. Thus the correspondence of a real number to a point is not one-to-one.

Some real numbers with the coordinates of their corresponding points are:

0 $(1, 0)$; $\pi/4$ $(\sqrt{2}/2, \sqrt{2}/2)$; $\pi/2$ $(0, 1)$; $3\pi/4$ $(-\sqrt{2}/2, \sqrt{2}/2)$; π $(-1, 0)$; $3\pi/2$ $(0, -1)$; 2π $(1, 0)$.