Trig Identity Problems

Math 130 *Kovitz*

1. A right triangle has an acute angle θ with $\sec\theta = \frac{1}{2}\sqrt{6}$.

   Find the exact values of $\sin\theta$, $\tan\theta$, $\csc\theta$, $\sin 2\theta$, and $\cos 2\theta$.

   All answers should be exact—in radical form. A decimal approximation receives no credit.

2. A right triangle has an acute angle θ with $\csc\theta = \sqrt{5}$.

   Find the exact values of $\cot\theta$, $\sec\theta$, and $\cos 2\theta$.

   All answers should be exact—in radical form. A decimal approximation receives no credit.

3. A right triangle has an acute angle θ with $\sin\theta = \frac{2}{7}\sqrt{6}$.

   Find the exact values of $\sec\theta$, $\tan\theta$, and $\cos 2\theta$.

   First find the three answers in exact radical form. Then, for all four trig functions (including the given one), find an approximation accurate to three decimal places.

4. Find an expression equivalent to
   $$\frac{\cos\theta}{\cot\theta}$$
   that contains at most one trig function.

   You may assume that θ is not an integer multiple of π or of $\pi/2$.

5. Find an expression equivalent to
   $$\frac{\sin\theta}{\tan\theta}$$
   that contains at most one trig function.

   You may assume that θ is not an integer multiple of π or of $\pi/2$.

6. Find an expression equivalent to
   $$\frac{\tan\theta}{\sin\theta}$$
   that contains at most one trig function.

   You may assume that θ is not an integer multiple of π.

7. Find an expression equivalent to
   $$\frac{\tan\theta}{\sec\theta}$$
   that contains at most one trig function.

   You may assume that θ is not an odd multiple of $\pi/2$.

8. Find an expression equivalent to
   $$\frac{\csc\theta}{\cot\theta}$$
   that contains at most one trig function.

   You may assume that θ is not an integer multiple of π.

9. Find an expression equivalent to
   $$(1+\cot^2\theta)(\sec^2\theta - 1)$$
   that contains at most one trig function.

   You may assume that θ is not an integer multiple of π.

10. Find an expression equivalent to

$$\frac{1}{\cos 2\theta + 1} - \frac{1}{2}$$

that contains at most one trig function.

11. Find an expression equivalent to

$$\frac{2 \sin \theta \cos \theta}{2 \cos^2 \theta - 1}$$

that contains at most one trig function.

Answers.

1. With $\sec\theta = \frac{1}{2}\sqrt{6}$, we can make a representative triangle using the fact that the secant is hypotenuse over adjacent.

   Set: hypotenuse $= \sqrt{6}$ and adjacent $= 2$.

   From the Pythagorean Therem, opposite$^2 + 2^2 = (\sqrt{6})^2$, opposite$^2 = 2$, and opposite $= \sqrt{2}$.

   Then: $\sin\theta = \frac{\sqrt{2}}{\sqrt{6}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$.

   $\tan\theta = \frac{\sqrt{2}}{2}$.

   $\csc\theta = \frac{1}{\sin\theta} = \sqrt{3}$.

   $\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{1}{\sqrt{3}}\right)\left(\frac{2}{\sqrt{6}}\right) = \frac{4}{\sqrt{18}} = \frac{4}{3\sqrt{2}} = \frac{2}{3}\sqrt{2}$.

   $\cos 2\theta = 1 - 2\sin^2\theta = 1 - 2\left(\frac{1}{\sqrt{3}}\right)^2 = 1 - 2\left(\frac{1}{3}\right) = \frac{1}{3}$.

2. With $\csc\theta = \sqrt{5}$, we can make a representative triangle using the fact that the cosecant is hypotenuse over opposite.

   Set: hypotenuse $= \sqrt{5}$ and opposite $= 1$.

   From the Pythagorean Therem, adjacent$^2 + 1^2 = (\sqrt{5})^2$, adjacent$^2 = 4$, and adjacent $= 2$.

   Then: $\cot\theta = \frac{2}{1} = 2$.

   $\sec\theta = \frac{\sqrt{5}}{2}$.

   $\cos 2\theta = 2\cos^2\theta - 1 = 2\left(\frac{2}{\sqrt{5}}\right)^2 - 1 = 2\left(\frac{4}{5}\right) - 1 = \frac{8}{5} - 1 = \frac{3}{5}$.

3. With $\sin\theta = \frac{2}{7}\sqrt{6}$, we can make a representative triangle using the fact that the sine is opposite over hypotenuse.

   Set: hypotenuse $= 7$ and opposite $2\sqrt{6}$.

   From the Pythagorean Therem, adjacent$^2 + (2\sqrt{6})^2 = 7^2$, adjacent$^2 = 49 - 48$, and adjacent $= 1$.

   Then: $\sec\theta = \frac{7}{5} = 1.4$.

   $\tan\theta = \frac{2\sqrt{6}}{5} = \frac{2}{5}\sqrt{6}$.

   $\cos 2\theta = 2\cos^2\theta - 1 = 2\left(\frac{5}{7}\right)^2 - 1 = 2\left(\frac{25}{49}\right) - 1 = \frac{50}{49} - 1 = \frac{1}{49}$.

   The four trig ratios as three-decimal approximations in the order given are:

   0.700 (approx), 1.4 (exact), 0.980 (approx), and 0.020 (approx).

   The fact that the pairs add to 1 is a bit of a coincidence as to four decimals they do not.

4. $\dfrac{\cos\theta}{\cos\theta/\sin\theta} = \cos\theta \times \dfrac{\sin\theta}{\cos\theta} = \sin\theta.$

5. $\dfrac{\sin\theta}{\sin\theta/\cos\theta} = \sin\theta \times \dfrac{\cos\theta}{\sin\theta} = \cos\theta.$

6. $\dfrac{\sin\theta/\cos\theta}{\sin\theta} = \dfrac{\sin\theta}{\cos\theta} \times \dfrac{1}{\sin\theta} = \dfrac{1}{\cos\theta} = \sec\theta.$

7. $\dfrac{\sin\theta/\cos\theta}{\sec\theta} = \dfrac{\sin\theta}{\cos\theta} \times \dfrac{1}{\sec\theta} = \dfrac{\sin\theta}{\cos\theta} \times \cos\theta = \sin\theta.$

8. $\dfrac{1/\sin\theta}{\cos\theta/\sin\theta} = \dfrac{1}{\sin\theta} \times \dfrac{\sin\theta}{\cos\theta} = \dfrac{1}{\cos\theta} = \sec\theta.$

9. Use $1+\tan^2\theta = \sec^2\theta$ and $1+\cot^2\theta = \csc^2\theta$, when needed.

   So: $(\csc^2\theta)(\tan^2\theta) = \frac{1}{\sin^2\theta} \times \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta} = \sec^2\theta.$

10. Use $1+\tan^2\theta = \sec^2\theta$, $1+\cot^2\theta = csc^2\theta$, and $\cos 2\theta = 2\cos^2\theta - 1$, when needed.

    So: $\frac{1}{2\cos^2\theta} - \frac{1}{2} = \frac{1}{2}(\sec^2\theta - 1) = \frac{1}{2}\tan^2\theta = \frac{\tan^2\theta}{2}.$

11. Use $\dfrac{\sin\theta}{\cos\theta} = \tan\theta$, $\sin 2\theta = 2\sin\theta\cos\theta$, and $\cos 2\theta = 2\cos^2\theta - 1$, when needed.

    So: $\dfrac{2\sin\theta\cos\theta}{2\cos^2\theta - 1} = \dfrac{\sin 2\theta}{\cos 2\theta} = \tan 2\theta.$