Challenge Problem 4

(due April 17)

Math 130 *Kovitz* Spring 2020

Take an arbitrary point in the first quadrant of the unit circle and call it (a, b). Let a triangle be drawn with vertices at (a, b) and at the left-most and right-most points of the unit circle.

Use the left-most point of the entire unit circle, not just the first-quadrant part.

Prove that this is a right triangle. Do not give numeric values to a and b; just work with the letters and the fact that the point (a, b) is located on the unit circle. (What formula do a and b satisfy?)

Methods: choose one.

- Look at the slopes of the three sides of the triangle.
- Look at the lengths of the three sides of the triangle.
- Look at the area of the triangle in two different ways.

  First let the base be the diameter. Then let the base be either of the two other sides.
- *A purely geometric argument:* Next connect the point (a, b) with the origin. Here do not use a and b—look at the relationships of the angles.
- *Alternate geometric method:* Rotate the upper unit semicircle 180° and then draw a line segment from the point (a, b) to its reflected point.