More practice for Test 1: odd even

Math 130 *Kovitz* Spring 2020: the test is on Wednesday, March 11.

1. In each case decide whether the function with the given rule is even, odd, or neither. Then check by evaluating the function for the given numbers.

   (a) $f(x) = x^{2/3}$. Find $f(8)$ and $f(-8)$.

   (b) $g(x) = \sqrt{x+16}$. Find $g(4)$ and $g(-4)$.

   (c) $h(x) = \frac{1}{x} + x$. Find $h(1)$ and $h(-1)$.

   (d) $f(x) = x^2 - x$. Find $f(3)$ and $f(-3)$.

   (e) $g(x) = (x^2 - x)(x+1)$. Find $g(3)$ and $g(-3)$.

   (f) $h(x) = (x^2 - 4x^4)(x^3 + 1)$. Find $h(2)$ and $h(-2)$.

   (g) $f(x) = \sqrt{x^2 + 4}$. Find $f(\sqrt{5})$ and $f(-\sqrt{5})$.

   (h) $g(x) = \dfrac{\sqrt{x^2 - 1}}{x}$. Find $g(2)$ and $g(-2)$.

   (i) $h(x) = \left(\sqrt{x}\right)^2$, only when defined as real numbers. Find $h(9)$ and $h(-9)$.

   (j) $f(x) = \sqrt{x^2}$. Find $f(3)$ and $f(-3)$.

   (k) $g(x) = 16$. Find $g(16)$ and $g(-16)$.

   (l) $h(x) = 0$. Find $h(4)$ and $h(-4)$.

   (m) $f(x) = x^3$. Find $f(2)$ and $f(-2)$.

   (n) $g(x) = x^3 + 2$. Find $g(3)$ and $g(-3)$.

   (o) $h(x) = |x - 1|$. Find $h(1)$ and $h(-1)$.

   (p) $f(x) = |x^2 - 3|$. Find $f(2)$ and $f(-2)$.

   (q) $g(x) = |(x-2)^2 - c|$. Find $g(6)$ and $g(-6)$.

   (r) $h(x) = \dfrac{2x^5 - \sqrt[3]{x}}{\sqrt[5]{x} + 2x}$. Find $h(1)$ and $h(-1)$.

   (s) $f(x) = (x-1)^3$. Find $f(4)$ and $f(-4)$.

   (t) $g(x) = \sqrt{x}\sqrt{x}$. Find $g(9)$ and $g(-9)$.

   The following problem is tricky. Usual check is not recommended; just use algebraic simplification to help to decide.

   (u) $h(x) = \left[3x - \left(\frac{1+\sqrt{3}}{5}\right)\right]\left[3x + \left(\frac{1+\sqrt{3}}{5}\right)\right]$.

Answers follow.

Answers.

1. In the cases where all x in the formula on the right side are to even powers, the function is even.

   (a) $f(x)$ is even. Rewrite as $\left(x^2\right)^{1/3}$ and use the rule of even powers. $f(8) = 4$ and $f(-8) = 4$.

   (b) $g(x)$ is neither. $g(4) = \sqrt{20} = 2\sqrt{5}$ and $g(-4) = 4$.

   (c) $h(x)$ is odd. $h(1) = 2$ and $h(-1) = -2$.

   (d) $f(x)$ is neither. $f(3) = 6$ and $f(-3) = 12$.

   (e) $g(x)$ is neither. $g(3) = 24$ and $g(-3) = -12$.

   (f) $h(x)$ is neither. $h(2) = -540$ and $h(-2) = 420$.

   (g) $f(x)$ is even. $f\left(\sqrt{5}\right) = 3$ and $f\left(-\sqrt{5}\right) = 3$. Also, the rule of even powers applies.

   (h) $g(x)$ is odd. $g(2) = \sqrt{3}/2$ and $g(-2) = -\sqrt{3}/2$.

   (i) $h(x)$ is neither. Its domain is $[0, \infty)$. which is unbalanced, so it cannot be even and it cannot be odd. $h(9) = 9$ but $h(-9)$ is not defined, because -9 is not in the domain..

   (j) $f(x)$ is even. $f(3) = 3$ and $f(-3) = 3$. The formula is equivalent to $f(x) = |x|$. Or simply note from the original form that the rule of even powers applies.

   (k) $g(x)$ is even. $g(16) = 16$ and $g(-16) = 16$. Any constant function of the form $f(x) = c$ will be even. The rule of even powers applies because all x in the formula (the right side) are to even powers. Since there are no x, the statement is true. Example: all men over 10 feet tall attend UMass Boston would be trivially true.

   (l) $h(x) = 0$ is even and $h(x) = 0$ is also odd. $h(4) = 0$ and $h(-4) = 0$, so it is easy to see that it is even by the rule inferred from the previous problem. However since $-0 = 0$, we can also say that $h(-a) = -h(a)$. That is in addition to the fact that $h(-a) = h(a)$. 'Both' only happens when the output of the function is always 0.

   (m) $f(x)$ is odd. $f(2) = 8$ and $f(-2) = -8$.

   (n) $g(x)$ is neither. Find that $g(3) = 29$ and $g(-3) = -25$. Because $-25 \neq 29$ and $-25 \neq -(29)$, it is verified that the function g is not odd and not even.

   (o) $h(x)$ is neither. $h(1) = 0$ and $h(-1) = 2$.

   (p) $f(x)$ is even. $f(2) = 1$ and $f(-2) = 1$. Also, the rule of even powers applies here.

   (q) $g(x)$ is neither. $g(6) = 16 - c$ and $g(-6) = 64 - c$. It is impossible for $g(-6)$ to ever be equal to $g(6)$. But, if c were equal to 40, $g(-6)$ and $-g(6)$ would be equal, and you'd need to pick another value besides 6 in order to show that $g(a)$ and $g(-a)$ are not equal for all real numbers. The rule of even powers does not apply here, because the power 2 is not a direct power of the variable x.

   (r) $h(x)$ is even. $h(1) = 1/3$ and $h(-1) = 1/3$. To substantiate, multiply top and bottom by x. The rule of even powers will then apply, because $x(\sqrt[3]{x}) = x^{4/3} = (x^4)^{1/3}$, and $x(\sqrt[5]{x}) = x^{6/5} = (x^6)^{1/5}$.

   (s) $f(x)$ is neither. $f(4) = 27$ and $f(-4) - 125$.

   (t) $g(x)$ is neither. $g(9) = 9$ but $g(-9)$ is not defined over the reals, because $\sqrt{(-9)}$ is not real. Or just note that the domain is $[0, \infty)$, which is not balanced. It's exactly the same problem as in part (i).

   (u) $h(x) = 9x^2 - \left(\frac{1+\sqrt{3}}{5}\right)^2$, so it is even by the rule of even powers..