Practice for Test 1

Math 130 *Kovitz* Spring 2020: the test is on Wednesday, March 11.

1. Find the slope-intercept equation for the straight line through the points $(3, 13)$ and $(12, 7)$. Then draw the graph and plot both given points and both intercepts, labelling each of them with their coordinates.

2. (a) Decide which three of the following four points are collinear and give algebraic justification.

$$A(1, 11), \quad B(9, 6), \quad C(13, 2), \quad D(25, -4)$$

   (b) Then find an equation of the straight line through the fourth point that is perpendicular to the line through the other three.

3. When $f(x) = 3x^2 - 5x - 11$ and $h \neq 0$, find $\dfrac{f(2+h) - f(2)}{h}$ and simplify the result.

4. In each case decide whether the function with the given rule is even, odd, or neither.

   (a) $f(x) = \dfrac{1}{x-1} + \dfrac{1}{x} + \dfrac{1}{x+1}$

   (b) $g(x) = \dfrac{x}{|x|} + x$

   (c) $h(x) = -\dfrac{1}{x} - x$

   (d) $i(x) = \dfrac{1}{x} - x$

   (e) $j(x) = |x + 1|$

   (f) $f(x) = |x + 3| - |x - 3|$

   (g) $g(x) = |x + 3| + |x - 3|$

   (h) $h(x) = \dfrac{|x+3| - |x-3|}{|x+3| + |x-3|}$

   (i) $i(x) = \sqrt{x+1} + \sqrt{1-x} + |x|$

   (j) $j(x) = \dfrac{x^2 - 3x}{|x|}$

   (k) $k(x) = \dfrac{1}{x^2 - 3} - 11x^6$

   (l) $f(x) = \dfrac{1}{x} + 2x$

   (m) $g(x) = x^3 - \dfrac{1}{x}$

   (n) $h(x) = \dfrac{x^3 + x}{\sqrt[3]{x}}$

   (o) $i(x) = \dfrac{x^4 - 1}{\sqrt[3]{x} - x}$

   (p) $j(x) = |x - 1|$

   (q) $f(x) = \sqrt{x^2 + 2}$

   (r) $g(x) = -|-x|$

   (s) $h(x) = \sqrt{x - 4}$. First determine the domain of h.

   (t) $i(x) = \dfrac{x}{x-1} - \dfrac{x^2}{x-1}$. First determine the domain of i.

   (u) $j(x) = \dfrac{1}{x^2 - 1} + \dfrac{1}{x+1}$.

5. Complete the square of
$$y = -\frac{1}{5}x^2 + 5x - 20,$$
getting the equation into the form
$$y = a(x-h)^2 + k.$$
What are the values of h and k?

Check by applying the formulas for h and k in terms of a, b, and c.

6. Consider the quadratic function
$$y = 2x^2 + 2x + 1.$$
   - Complete the square.
   - State—as an ordered pair—the coordinates of the vertex of the graph.
   - Write the equation of the line of symmetry.
   - Solve the equation $0 = 2x^2 + 2x + 1$.
   - State—as ordered pairs—the coordinates of the y-intercept of the graph and of its symmetric partner.
   - State—as ordered pairs—the coordinates of the x-intercepts (if any) of the graph.
   - Plot, with coordinates, one point in each quadrant where you have not yet plotted a point.
   - **Graph the equation,** using the points you found above.

7. Consider the quadratic function
$$y = 2x^2 + 5x + 3.$$
   - Complete the square.
   - State—as an ordered pair—the coordinates of the vertex of the graph.
   - Write the equation of the line of symmetry.
   - Solve the equation $0 = 2x^2 + 5x + 3$.
   - State—as ordered pairs—the coordinates of the y-intercept of the graph and of its symmetric partner.
   - State—as ordered pairs—the coordinates of the x-intercepts (if any) of the graph.
   - Plot, with coordinates, one point in each quadrant where you have not yet plotted a point.
   - **Graph the equation,** using the points you found above.

8. Find two positive real numbers whose product is a minimum, under the condition that the second number is 15 smaller than the first.

   What is that minimum product?

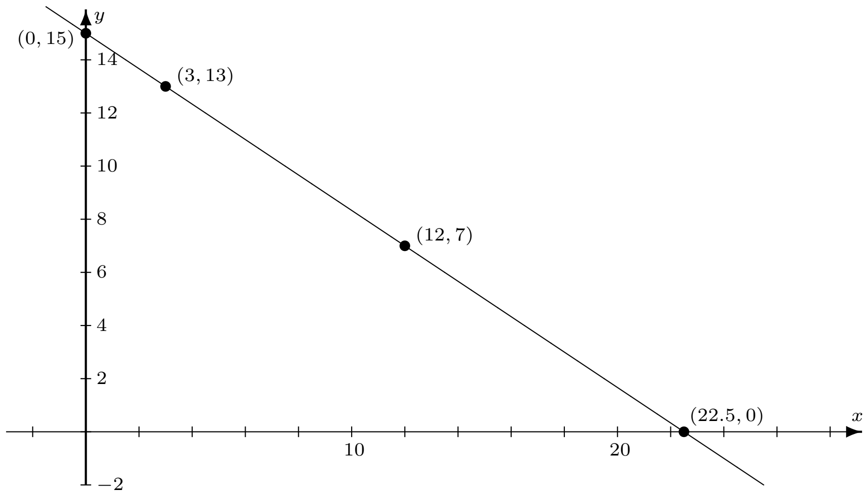
9. Find two positive real numbers whose product is a maximum, under the condition that the sum of five times the first number and twice the second number is 20.

   What is that maximum product?

Answers follow.

Answers.

1. $y = -\dfrac{2}{3}x + 15.$



2. (a) A, B and D are colinear.
   (b) $y = 1.6x - 18.8$.

3. $$\frac{3(2+h)^2 - 5(2+h) - 11 - [3\cdot 2^2 - 5(2) - 11]}{h} = \frac{3(4+4h+h^2) - 10 - 5h - 11 - [-9]}{h} =$$
$$\frac{12 + 12h + 3h^2 - 10 - 5h - 11 + 9}{h} = \frac{3h^2 + 7h}{h} = 3h + 7.$$

4. (a) $f(x)$ is odd.
   (b) $g(x)$ is odd.
   (c) $h(x)$ is odd.
   (d) $i(x)$ is odd.
   (e) $j(x)$ is neither odd nor even.
   (f) $f(x)$ is odd.
   (g) $g(x)$ is even.
   (h) $h(x)$ is odd.
   (i) $i(x)$ is even.
   (j) $j(x)$ is neither odd nor even.
   (k) $k(x)$ is even.
   (l) $f(x)$ is odd.
   (m) $g(x)$ is odd.
   (n) $h(x)$ is even.
   (o) $i(x)$ is odd.
   (p) $j(x)$ is neither odd nor even.
   (q) $f(x)$ is even.
   (r) $g(x)$ is even.
   (s) $h(x)$ is neither odd nor even, because the domain $[4, \infty)$ is not balanced.
   (t) $i(x)$ must be neither odd nor even, because the domain (all reals except 1) is unbalanced. This is in spite of the fact that the formula is $i(x) = -x$ over the domain of definition. That tells us that $i(-x) = -(-x) = x = -i(x)$ whenever both x and $-x$ are in the domain of i. The case where $x = -1$ should lead to $i(-(-1)) = i(1)$, but $i(1)$ is not defined.
   The equality $i(-a) = -i(a)$ does not hold for all a in the domain of i, so the function is not odd.

(u) $j(x)$ is odd. The domain is all reals except -1 and 1, which is balanced. The formula simplifies to $j(x) = \frac{x}{x^2-1}$, which is seen to be odd.

If you are still skeptical, try finding $j(2)$ and $j(-2)$, getting $\frac{1}{4-1} + \frac{1}{2+1} = 2/3$ and $\frac{1}{(-2)^2-1} + \frac{1}{-2+1} = \frac{1}{3} + \frac{1}{-1} = \frac{1}{3} - 1 = -2/3$. For this choice, $-j(a)$ equals $j(-a)$, supporting the finding of odd.

5. $y = -\dfrac{1}{5}x^2 + 5x - 20.$

$y = -\dfrac{1}{5}(x^2 - 25x) - 20.$

$y = -\dfrac{1}{5}\left[x^2 - 25x + 625/4 - 625/4\right] - 20.$

$y = -\dfrac{1}{5}\left[(x - 25/2)^2 - 625/4\right] - 20.$

$y = -\dfrac{1}{5}(x - 25/2)^2 - \dfrac{1}{5}(-625/4) - 20.$

$y = -\dfrac{1}{5}(x - 25/2)^2 + 125/4 - 20.$

$y = -\dfrac{1}{5}(x - 25/2)^2 + 45/4.$

The values of (h, k) are $(12.5, 11.25)$.

Start with $a = -1/5$, $b = 5$, and $c = -120$.

It gives
$h = (-5)/(-2/5) = 25/2$ and
$k = c - b^2/4a = -20 - 25/(-4/5) = -20 + 125/4 = 45/4.$

6. Consider the quadratic function

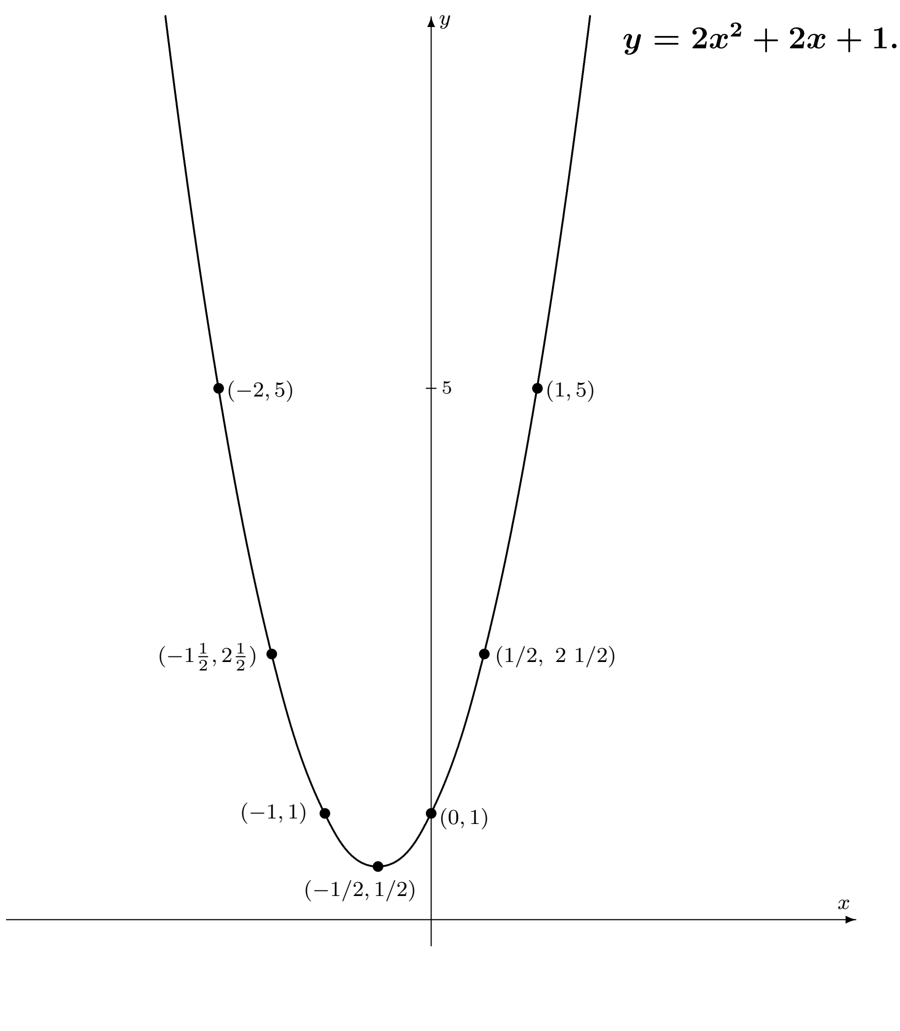
$$y = 2x^2 + 2x + 1.$$

- $y = 2(x^2+x)+1 = 2[x^2+x+1/4-1/4]+1 = 2(x+1/2)^2-1/2+1 = 2(x+1/2)^2+1/2.$

  Key Steps in Completing the Square:

  (a) Factor out the 2 by dividing the term $2x$ by 2 to get x.
  (b) Determine the completing number. From x, get 1, then 1/2, then $(1/2)^2 = 1/4$.
  (c) Replace the parentheses with square brackets.
  (d) Add and subtract 1/4 *inside* the square brackets.
  (e) Replace the first three terms inside with the perfect square $(x + 1/2)^2$.
  Combine the initial letter with the third step from the completing process above. So (x, then 1/2) will give $(x + 1/2)$.
  (f) Distribute to the dangling term. So $2(-1/4)$ becomes $-1/2$.
  (g) Combine the two constants: $-1/2 + 1 = 1/2$.

- $(-0.5,\ 0.5)$.
- $x = -0.5$.
- $x = \dfrac{-2 \pm \sqrt{4-4(2)(1)}}{4} = \dfrac{-2 \pm \sqrt{-4}}{4}$, so there are no real solutions.
- $(0, 1)$ and $(-1, 1)$.
- There are no x-intercepts.
- The misssing quadrant is the first quadrant and two points are: $(1/2, 2\frac{1}{2})$ and $(1, 5)$.



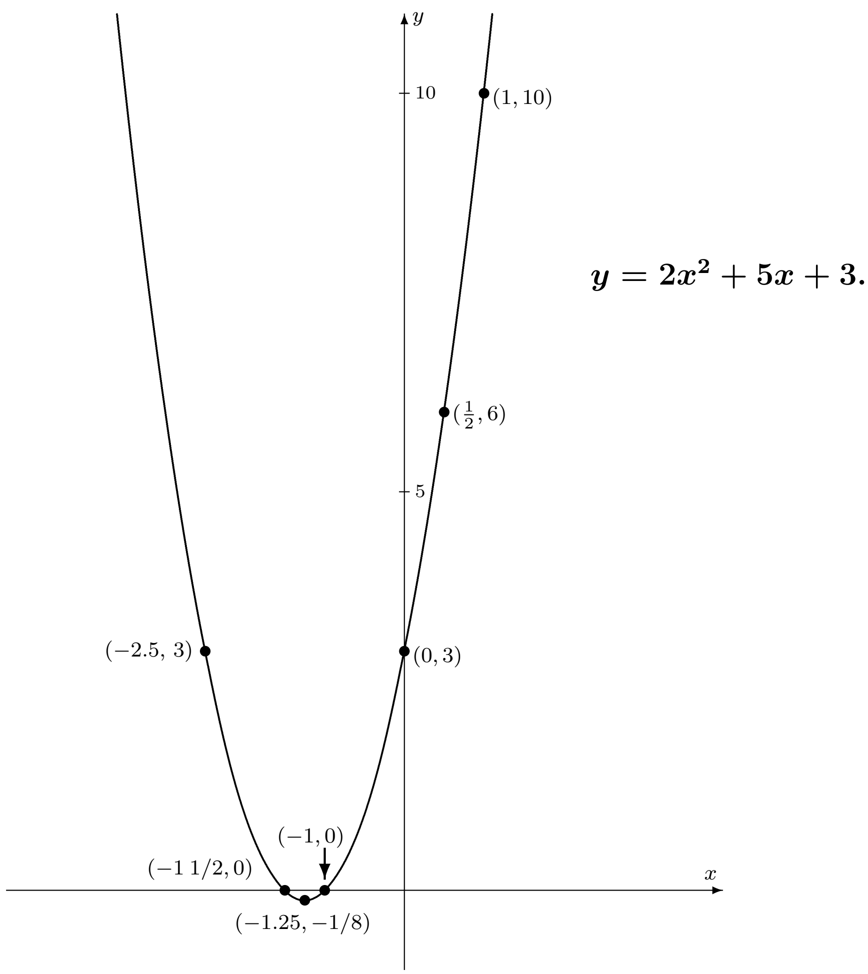
7. Consider the quadratic function

$$y = 2x^2 + 5x + 3.$$

- $y = 2(x^2 + \frac{5}{2}x) + 3 = 2[x^2 + \frac{5}{2}x + 25/16 - 25/16] + 3 = 2(x + 5/4)^2 - 2(25/16) + 3 = 2(x + 5/4)^2 - 25/8 + 3 = 2(x + 5/4)^2 - 1/8.$

  Key Steps. Factor out $5x/2$; Get completing no.: $\frac{5}{2}x$ to $5/2$ to $5/4$ to $(5/4)^2 = 25/16$; Add and subtract $25/16$ inside the square brackets; Create the perfect square from x and $5/4$, and replace first three terms: $(x + 5/4)^2$; Distribute to the dangling term: $2(-25/16) = -25/8$; Combine the constants: $-25/8 + 3 = -25/8 + 24/8 = -1/8$.

- $(-1.25,\ -0.125)$.
- $x = -1\,1/4$ or $x = -1.25$.
- $x = \dfrac{-5 \pm \sqrt{25 - 4(2)(3)}}{4} = \dfrac{-5 \pm \sqrt{1}}{4}$. $x = -1$ or $x = -1.5$.
- $(0, 3)$ and $(-2.5, 3)$.
- $(-1.5, 0)$ and $(-1, 0)$.
- The misssing quadrant is the first quadrant and two points are: $(1/2,\ 6)$ and $(1, 10)$.



8. Call the first number x, the second number z and the product p. Do not use the variable y for the second number; it is not the output and calling it y could lead to massive confusion. The product, however, could have been designated by y instead of p.

   The formula for the product, $p = xz$, can be simplified to two variables if the condition that $z = x - 15$ is used to substitute for z.

   This gives $p = x(x - 15) = x^2 - 15x$. So p is a funcion of x.

   The input variable is x and the output variable is p. This parabola opens up and has a minimum at the vertex.

   Complete the square: $p = x^2 - 15x + (15/2)^2 - (15/2)^2 = x^2 - 15x + 225/4 - 225/4 = (x - 7.5)^2 - 56.25$.

   The vertex is at $(7.5, -56.25)$.

   To find the second number, z, use $z = x - 15 = 7.5 - 12 = -7.5$.

   The two numbers are 7.5 and -7.5, and the mimimum product is -56.25.

   Shortcut OK. $a = 1$, $b = -15$, $c = 0$. Then: $h = 15/(2(1))$ and $k = 0 - (-15)^2/(4(1)) = -225/4 = -56.25$.

9. $5x + 2w = 20$

   $w = -\frac{5}{2}x + 10$

   Substitute into $p = wx$ to get $p = \left(-\frac{5}{2}x + 10\right) x = -\frac{5}{2}x^2 + 10x$.

   This is maximized when $x = -b/2a = \frac{-10}{-5} = 2$.

   Because $5 \cdot 2 + 2w = 20$, $w = 5$.

   The numbers are 2 and 5; and their product is 10. That is at the vertex of a parabola that opens down, so it is a maximum product, the product being the output of the function $p = f(x)$.