Sample of Typical Final Examination Problems

Math 130 Precalculus for the Dec. ??, 2019 Final Exam

No books, notes, or graphing calculators; scientific calculators permitted.
Show all work with at least four-decimal-place accuracy.

1. When $f(x) = \dfrac{1}{x+1}$ and $h \neq 0$, find $\dfrac{f(2+h) - f(2)}{h}$ and simplify the result.

2. In each case decide whether the function with the given rule is even, odd, or neither. Explain your reasoning or support your answer.

   (a) $f(x) = \dfrac{1}{x-1} + \dfrac{1}{x} + \dfrac{1}{x+1}$

   (b) $g(x) = |x+1|$

   (c) $h(x) = |x+3| - |x-3|$

   (d) $i(x) = \sqrt{(4-x)(4+x)}$

   (e) $j(x) = \sqrt{x+1} \cdot \sqrt{x-1}$, defined for real-valued outputs only.

3. Let $f(x) = \sqrt{x}$ for all $x \geq 0$ and $g(x) = x^2$ for all real numbers.

   True or false:

   (a) $(f \circ g)(x) = x$ for all real numbers.

   (b) f and g are inverse functions.

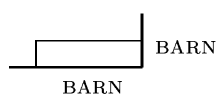
4. Determine whether $f(x) = \dfrac{4}{-5x+3}$ has an inverse function. If it does, find the inverse function.

   If it has an inverse function:

   What is the value of $f(1)$? Call it a.

   Apply the function f inverse (written as f^{-1}) to a. Is the result OK?

5. Find the largest area that a farmer can enclose by constructing a rectangular pen from 26 feet of fencing, if he uses a corner of his barn for two walls of the pen.



6. On the graph of the function $y = \log_2 x$, when the x-coordinate of point B is 16 times the square of the x-coordinate of point A, how is the y-coordinate of point B related to the y-coordinate of point A?

7. Find

   (a) $4^{\log_4 1/2}$.

   (b) $\log_4(4^{1/2})$.

8. Decide if each statement is true or false. Then justify your answer by writing an equation.

   (a) Multiplying two numbers then taking the log gives the same result as taking each log and then adding them.

   (b) Taking the logs of two numbers then dividing those two logs gives the same result as subtracting the two numbers then taking the log of that difference.

   (c) The product of $\log_a b$ and $\log_b a$ is always equal to 1.

9. In each of parts (a) through (h), approximate the logarithm, using the properties of logarithms, given $\log_b 2 \approx 1.098$, $\log_b 3 \approx 1.740$, and $\log_b 5 \approx 2.5495$.

   (a) $\log_b 6$
   (b) $\log_b \frac{3}{5}$
   (c) $\log_b 125$
   (d) $\log_b \sqrt{3}$
   (e) $\log_b 20$
   (f) $\log_b (4b)^{-2}$
   (g) $\log_b (5b^2)$
   (h) $\log_b \sqrt[3]{2b}$

10. Solve algebraically:

    (a) $\log x + \log(x - 15) = 2.$
    (b) $\log x - \log(x - 15) = 2.$
    (c) $\log 24x - \log(1 + \sqrt{x}) = 2.$

11. First decide in which intervals all valid solutions must lie.

    Then solve for x.
    $$\log_2 x - \log_2(2x - 1) = -3.$$
    Check your solutions in the original equation.

12. Solve algebraically: $\log_2 x + \log_2(1 - 3x) = -4.$

    Hint: to solve the equation found after applying some log rules, just use the quadratic formula.

13. First decide in which intervals all valid solutions must lie.

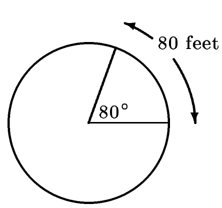
    Then solve for x.
    $$\log_2 x + \log_2(1 - 2x) = -3.$$
    Check your solutions in the original equation.

14. Solve for x:

    (a) $\dfrac{\log_3 x}{\log_3(x - 1)} = 2.$
    (b) $\log_3 x - \log_3(x - 1) = 2.$

15. (a) Rewrite in radian measure as a fractional multiple of π and in degree measure: $3/16$ of a revolution.
    (b) Rewrite in degree measure: $\dfrac{7\pi}{8}$.
    (c) Rewrite in radian measure as a fractional multiple of π in lowest terms: $132°$.
    (d) Find the length of the arc on a circle of radius $150/\pi$ feet intercepted by a central angle of $150°$.

16. A group has a banner that is 80 feet long. They wish to display it in the form of an arc of a circle that has angular measure of $80°$. What is the radius of the circle needed for this layout? The answer may be left as $\frac{N}{\pi}$ feet.



17. A right triangle has an acute angle θ with $\sec\theta = \frac{8}{7}$. Find the exact values of the other five trigonometric functions of θ, in fractional form. Some of the expressions will involve square roots; do not convert the square roots to decimals.

    Then find the exact values of $\sec(90° - \theta)$ and of $\csc^2\theta - 1$, also in fractional form.

    Hint. First sketch a right triangle corresponding to that secant. Next use the Pythagorean Theorem to determine the third side. Then find the other five trigonometric functions of θ.

    For the other two values, use the appropriate trigonometric identities.

18. In each part, decide whether the identity is true or false. If true, verify it.

    (a) $\csc x - \sin x = \cot x \cos x$.

    (b) $\dfrac{1}{\cos x} - \dfrac{1}{\sec x} = \cos x - \sec x$.

    (c) $(1 + \cot x)^2 = \csc^2 x$.

19. Simplify and reduce to an expression that contains at most one trig function.

    (a) $\cos x(1 + \tan x)(1 - \tan x)$

    (b) $\tan x \cos^2 x$

    (c) $\cos^4 x - \sin^4 x$

    (d) $\dfrac{1 + \cot^2 x}{\sin x}$

    (e) $\dfrac{\sec x}{\csc x}$

    (f) $\dfrac{\sec x}{\sin x}$

20. Find all solutions to $\sin\theta = 0.669$ in the interval $0 \le \theta < 180°$.

    Round off both answers to the nearest 0.01°.

21. Find all solutions to $\cos\theta = -0.26$ in the interval $0 \le \theta < 360°$.

    Round off both answers to the nearest 0.01°.

22. (a) Find all solutions with $0 \le x \le 2\pi$ for $\sin 2x = -\cos x$.

    (b) Graph $\sin 2x$ and $-\cos x$ on the same axes and indicate on your sketch the points corresponding to the solutions in part (a).

23. (a) Find all x between 0 and 2π for which $\sin 2x = -\sin x$.

    (b) Sketch the graphs of $\sin 2x$ and $-\sin x$ on the same axes, indicating on your sketch the points corresponding to the solutions in part (a).

24. Find the period and the amplitude of

$$y = 5\sin\left(2x - \frac{\pi}{4}\right).$$

    Graph one period. Label with coordinates the endpoints of that period, the highest and lowest points, and all intercepts in that period.

    State the phase fraction: the portion of a period that the graph was translated right (+) or left (−).

    It might be less confusing with the 2 factored out of the expression in the parentheses.

25. Write an algebraic expression that is equivalent to the given expression.

$$\cos\left(\arcsin\frac{1}{x}\right)$$