Sample Final Examination

Math 130 Precalculus of the Dec. ??, 2019 Final Exam

No books, notes, or graphing calculators; scientific calculators permitted.

Show all work with at least four-decimal-place accuracy.

1. Find an equation of the line passing through the points $(8, -1)$ and $(-2, 3)$. Sketch the line and label both intercepts with their coordinates.

2. Write the standard form of the equation of the circle for which the endpoints of a diameter are $(-2, -4)$ and $(3, 8)$.

3. When $f(x) = 2x^2 + 3x - 1$ and $h \neq 0$, find $\dfrac{f(2+h) - f(2)}{h}$ and simplify the result.

4. In each part, how is the graph of the given function related to the graph of the parent function $f(x) = \sqrt{x}$?

    (a) $g(x) = \sqrt{x+5} + 5$.

    (b) $h(x) = 2\sqrt{x-3}$.

    (c) $i(x) = -\sqrt{-x-3}$.

5. For $f(x) = \sqrt{x-4}$ and $g(x) = x^2 + 4$,

    (a) Find $f \circ g$ and $g \circ f$.

    Find the domain of each composite function (the domain of $f \circ g$ and of $g \circ f$).

    (b) Find the domain and the range of f.

    Find the domain and the range of g.

    In light of the domain and range of f and g, decide whether f and g are true inverse functions of each other.

6. The function f is described by the equation $f(x) = \sqrt{x+1}$ and the domain $[-1, \infty)$.

    (a) What is the range of f?

    What are the coordinates of its x and y-intercepts?

    (b) Is the function f one-to-one?

    If so, find its inverse, giving the formula and the domain.

    What are the coordinates of the x and y-intercepts of the inverse function of f?

7. Complete the square, getting the equation $y = \frac{1}{4}x^2 - 2x - 12$ into standard form, and sketch its graph.

    Plot the vertex, axis of symmetry, and all intercepts, labelling with both coordinates or the equation.

8. Consider ordered pairs of real numbers for which one fourth of the first number plus the second number equals 5. Of all such ordered pairs, which one has the largest product?

    For that maximizing pair, find the first and second numbers and their product.

9. For the function $f(x) = 2^x$, decide whether each of the following statements is true or false.

    (a) If you change the sign of the input, the output ends up the reciprocal of what it was previously.

    (b) If you double the input, the output ends up the square of what it was previously.

    (c) If you add 3 to the input, the output ends up 8 times as much as it was previously.

    (d) If you square the input, the new output is the old output raised to the previous input.

10. Given that $\log_b 2 = 3/4$, find $\log_b \frac{1}{2}$, $\log_2 b$, and (to two decimal places) b.

11. True or false:

    (a) $\log(3.4 \times 13.4) = \log 3.4 + \log 13.4$.

    (b) $\log 2.5 \times \log 4 = \log 6.5$.

    (c) $\dfrac{\log \frac{1}{2}a - \log a}{\log \left(\frac{1}{2}\right)^{1/2}} = 2$ for all a such that $a > 0$.

    (d) $\log \left(\frac{1}{100}a^2\right) = 2 \log a - 2$ for a any non-zero real number.

    (e) The log of the quotient equals the difference of the logs.

    (f) The log of the sum equals the product of the logs.

12. True or false:

    (a) $\log(10.1 + 10,000,000) = (\log 10.1)(\log 10,000,000)$.

    (b) $\dfrac{\frac{1}{2}\log a}{\log \frac{1}{2}a} = 1$ for all a such that $a > 0$, $a \neq 1$, and $\neq 2$.

    (c) $\log \left(\frac{1}{2}x^2\right) = 2\log|x| - \log 2$.

    (d) The log of the sum equals the product of the logs.

    (e) The numerical value of $\log_a b$ is the reciprocal of the numerical value of $\log_b a$.

13. True or false:

    (a) $\dfrac{\log_a 5}{\log_a 2} = \log_2 5$.

    (b) $\dfrac{\log_a 5}{\log_a 2} = \dfrac{\log 5}{\log 2}$.

14. Let $\log_a 2 = 0.2$.
    Find.
    (a) $\log_a 32 + \log_a \frac{1}{4}$.
    (b) $\log_a 4\sqrt{2}$.
    (c) A 4-decimal-place approximation of $\log_a 5$ (calculator needed).

15. Solve the equation $\log x - \log(x-1) = 2$.

    Before solving, decide which x-values are valid substitutions into both of these logs.

16. A population of fruit flies is increasing according to the law of exponential growth. At time 2 hours there are 2 pounds of flies and at time 32 hours there are 32 pounds of flies.

    (a) Find the exact value of the doubling time. (No calculator is necessary.)

    (b) True or false: at time 8 hours there were exactly 8 pounds of fruit flies.

    (c) If false, about how many pounds of fruit flies were there at time 8 hours (to the nearest three-decimal accuracy or as an exact radical expression).

    (d) At exactly what time will there be 64 pounds of fruit flies?

17. A right triangle has an acute angle θ with $\csc\theta = 5/2$.

    Use the appropriate trigonometric identities to find:

    (a) $\sin\theta$.
    (b) $\cot\theta$.
    (c) $\cos 2\theta$.
    (d) $\sin(30° + \theta)$.

18. Find the hypotenuse of a right triangle in which an acute angle of $30°$ has an adjacent leg of 4 inches.

19. Find all solutions to $\cos\theta = \sqrt{3}/2$ in the interval $0 \le \theta < 2\pi$.

20. Find the exact value of $\sin 165°$.

21. Solve for x: $\log\left(\frac{25x}{4}\right) - \log(5 + \sqrt{x}) = 2$.

22. (a) Find all solutions with $-90 \le x < 90°$ for $\csc 2x = 1 + \tan^2 x$.

         i. Give the solution(s) in terms of the arctangent function.

         ii. Give each answer in degrees, approximated to the nearest 0.00001 degree.

    (b) Verify the solution(s) by plugging in the approximate angle(s) into the original formula and using a calculator.