Sample Final Examination

Math 130 Precalculus for the May 22, 2019 Final Exam

No books, notes, or graphing calculators; scientific calculators permitted.

Show all work with at least four-decimal-place accuracy.

1. Find an equation of the line passing through the points $(8, -1)$ and $(-2, 3)$. Sketch the line and label both intercepts with their coordinates.

2. When $f(x) = \dfrac{1}{x+1}$ and $h \neq 0$, find $\dfrac{f(2+h) - f(2)}{h}$ and simplify the result.

3. In each case decide whether the function with the given rule is even, odd, or neither. Explain your reasoning or support your answer.

   (a) $f(x) = \dfrac{1}{x-1} + \dfrac{1}{x} + \dfrac{1}{x+1}$

   (b) $g(x) = |x+1|$

   (c) $h(x) = |x+3| - |x-3|$

   (d) $i(x) = \sqrt{(4-x)(4+x)}$

   (e) $j(x) = \sqrt{x+1} \cdot \sqrt{x-1}$, defined for real-valued outputs only.

4. In each part, how is the graph of the given function related to the graph of the parent function $f(x) = \sqrt{x}$?

   (a) $g(x) = \sqrt{x+5} + 5$.

   (b) $h(x) = 2\sqrt{x-3}$.

   (c) $i(x) = -\sqrt{-x-3}$.

5. For $f(x) = \sqrt{x-4}$ and $g(x) = x^2 + 4$,

   (a) Find $f \circ g$ and $g \circ f$.

   Find the domain of each composite function (the domain of $f \circ g$ and of $g \circ f$).

   (b) Find the domain and the range of f.

   Find the domain and the range of g.

   In light of the domain and range of f and g, decide whether f and g are true inverse functions of each other.

6. Determine whether $f(x) = \dfrac{4}{-5x+3}$ has an inverse function. If it does, find the inverse function.

   If it has an inverse function:

   What is the value of $f(1)$? Call it a.

   Apply the function f inverse (written as f^{-1}) to a. Is the result OK?

7. Complete the square, getting the equation $y = \frac{1}{4}x^2 - 2x - 12$ into standard form, and sketch its graph. Plot the vertex, axis of symmetry, and all intercepts, labelling with both coordinates or the equation.

Log Problems are on the next page.

8. On the graph of the function $y = \log_2 x$, when the x-coordinate of point B is 16 times the square of the x-coordinate of point A, how is the y-coordinate of point B related to the y-coordinate of point A?

9. Given that $\log_b 2 = 3/4$, find $\log_b \frac{1}{2}$, $\log_2 b$, and (to two decimal places) b.

10. True or false:

    (a) $\log(3.4 \times 13.4) = \log 3.4 + \log 13.4$.

    (b) $\log 2.5 \times \log 4 = \log 6.5$.

    (c) $\dfrac{\log \frac{1}{2}a - \log a}{\log \left(\frac{1}{2}\right)^{1/2}} = 2$ for all a such that $a > 0$.

    (d) $\log \left(\frac{1}{100}a^2\right) = 2 \log a - 2$ for a any non-zero real number.

    (e) The log of the quotient equals the difference of the logs.

    (f) The log of the sum equals the product of the logs.

11. Solve the equation $\log x - \log(x-1) = 2$.

    Before solving, decide which x-values are valid substitutions into both of these logs.

12. Solve algebraically:

    (a) $\log x + \log(x-15) = 2$.

    (b) $\log x - \log(x-15) = 2$.

    (c) $\log 24x - \log(1+\sqrt{x}) = 2$.

13. Solve for x:

    (a) $\dfrac{\log_3 x}{\log_3(x-1)} = 2$.

    (b) $\log_3 x - \log_3(x-1) = 2$.

14. A population of fruit flies is increasing according to the law of exponential growth. At time 2 hours there are 2 pounds of flies and at time 32 hours there are 32 pounds of flies.

    (a) Find the exact value of the doubling time. (No calculator is necessary.)

    (b) True or false: at time 8 hours there were exactly 8 pounds of fruit flies.

    (c) If false, about how many pounds of fruit flies were there at time 8 hours (to the nearest three-decimal accuracy or as an exact radical expression).

    (d) At exactly what time will there be 64 pounds of fruit flies?

Trig Problems are on the next page.

15. (a) Rewrite in radian measure as a fractional multiple of π and in degree measure: 3/16 of a revolution.

    (b) Rewrite in degree measure: $\dfrac{7\pi}{8}$.

    (c) Rewrite in radian measure as a fractional multiple of π in lowest terms: $132°$.

    (d) Find the length of the arc on a circle of radius $150/\pi$ feet intercepted by a central angle of $150°$.

16. A right triangle has an acute angle θ with $\sec\theta = \frac{8}{7}$. Find the exact values of the other five trigonometric functions of θ, in fractional form. Some of the expressions will involve square roots; do not convert the square roots to decimals.

    Then find the exact values of $\sec(90° - \theta)$ and of $\csc^2\theta - 1$, also in fractional form.

    Hint. First sketch a right triangle corresponding to that secant. Next use the Pythagorean Theorem to determine the third side. Then find the other five trigonometric functions of θ.

    For the other two values, use the appropriate trigonometric identities.

17. Find the hypotenuse of a right triangle in which an acute angle of $30°$ has an adjacent leg of 4 inches.

18. In each part, decide whether the identity is true or false. If true, verify it.

    (a) $\csc x - \sin x = \cot x \cos x$.

    (b) $\dfrac{1}{\cos x} - \dfrac{1}{\sec x} = \cos x - \sec x$.

    (c) $(1 + \cot x)^2 = \csc^2 x$.

19. Find all solutions to $\cos\theta = \sqrt{3}/2$ in the interval $0 \le \theta < 2\pi$.

20. (a) Find all solutions with $0 \le x \le 2\pi$ for $\sin 2x = -\cos x$.

    (b) Graph $\sin 2x$ and $-\cos x$ on the same axes and indicate on your sketch the points corresponding to the solutions in part (a).

21. Find the exact value of $\sin 165°$.

22. Find the size of the angle between the sides of lengths 5 and 16 in a triangle with sides of 5, 16, and 19.

23. Write an algebraic expression that is equivalent to the given expression.

$$\cos\left(\arcsin\frac{1}{x}\right)$$