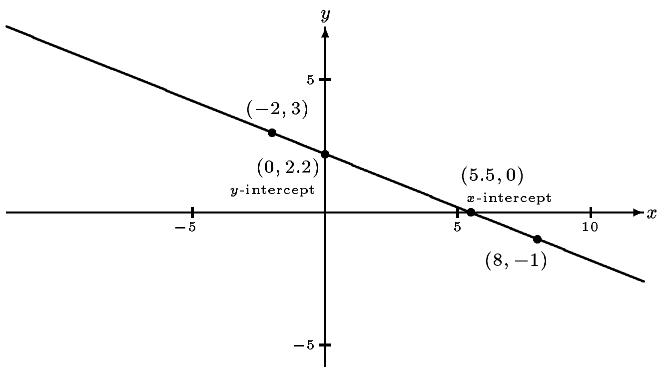
Answers to the Sample Examination

Math 130 Precalculus for the May 22, 2019 Final Exam

1. $y = -\frac{2}{5}x + \frac{11}{5}$ or $y = -0.4x + 2.2$

Also $y - 3 = -\frac{2}{5}(x + 2)$ or $y + 1 = -\frac{2}{5}(x - 8)$

Also $2x + 5y = 11$.



Work: $m = \frac{-1-3}{8-(-2)} = \frac{-4}{10} = -2/5$ and $y - 3 = -\frac{2}{5}(x + 2)$.

2. $$\frac{f(2+h) - f(2)}{h} = \frac{\frac{1}{3+h} - \frac{1}{3}}{h} = \frac{\frac{-h}{3(3+h)}}{h} = \frac{-1}{3(3+h)}$$

3. (a) f is odd because it simplifies to $\frac{3x^2-1}{x(x^2-1)}$, and when x has opposite sign, so does this fraction.
   (b) g is neither even nor odd because $g(3) = 4$ while $g(-3) = 2$.
   (c) h is odd because $h(a) = |a+3| - |a-3|$ and $h(-a) = |-a+3| - |-a-3| = |a-3| - |a+3| = -h(a)$. For support, show that $h(4) = 6$ and $h(-4) = -6$.
   (d) i is even because $i(x) = \sqrt{16 - x^2}$ showing that it is even by the rule of even powers and also that the domain is balanced, being $[-4, 4]$.
   (e) j is neither even nor odd because the domain, $x \geq 1$, is not balanced.

4. (a) The graph of $y = \sqrt{x}$ was moved 5 units to the left and 5 units up.
   (b) The graph of $y = \sqrt{x}$ was stretched vertically by a factor of 2 and moved 3 units to the right.
   (c) It is best to rewrite this as $i(x) = -\sqrt{-(x+3)}$.
   Then graph an intermediate function, $y = \sqrt{-x}$, first. Its graph is the graph of $f(x)$ reflected across the y-axis.
   Then replace x with $(x + 3)$ to get $y = \sqrt{-(x+3)}$. This moves the previous graph 3 units to the left.
   Finally, $i(x) = -\sqrt{-x-3}$ is just the previous graph reflected across the x-axis, since all y values will assume opposite sign.

   Answer: The graph of $y = \sqrt{x}$ was reflected across the y-axis, then shifted 3 units to the left, and then reflected across the x-axis.

   Verify by noting that the point $(9, 3)$ on f will end up as the point $(-12, -3)$ on i and that the point $(-12, -3)$ is indeed a solution of the equation for i because $-3 = -\sqrt{-(-12) - 3}$.

5. (a) $f \circ g(x) = f(g(x)) = \sqrt{(x^2 + 4) - 4} = \sqrt{x^2} = |x|$ and $g \circ f(x) = g(f(x)) = (\sqrt{x-4})^2 + 4 = x - 4 + 4 = x$.
   The domain of $f \circ g$ is all reals since g always has an output of 4 or more which, when input into f guarantees that the number under the square root will be positive.
   The domain of $g \circ f$ is $[4, \infty)$, as that is the domain of f.

(b) f: The domain is $[4,\infty)$ and the range is $[0,\infty)$.

g: The domain is all reals and the range if $[4,\infty)$.

Since the domain of g is not the same as the range of f, f and g are not inverse functions.

6. $f^{-1}(x) = \dfrac{3x-4}{5x}$. It was found by solving $x = \dfrac{4}{-5y+3}$ for y. It is a function.

As a verbal string it is divide by 4; take reciprocal; subtract 3; change sign; divide by 5. This is the formula

$$-\frac{1}{5}\left(\frac{4}{x}-3\right).$$

The two answers, while different formulas, are algebraically equivalent.

The orginal verbal string was (before inversion) multiply by 5, change sign, add 3, take reciprocal, multiply by 4. All that had to be done was to find the inverse of each operation and apply them in the opposite order as they were in the original function.

Because this problem is not easily conceptualized, one should apply the two results to a number of his choosing to see if they work.

Let $x=3$, so $f(3) = \frac{4}{-15+3} = \frac{4}{-12} = -\frac{1}{3}$.

Then $f^{-1}(-1/3) = \frac{3(-1/3)-4}{5(-1/3)} = \frac{-5}{-5/3} = 3$. It checks.

And the verbal: $-1/3$ divided by 4 is $-1/12$, reciprocal is -12, subtract 3 gives -15, change sign gives 15, divide by 5 gives 3, as expected.

$f(1) = -2$, so $a=-2$.

$f^{-1}(-2) = \frac{3(-2)-4}{5(-2)} = \frac{-10}{-10} = 1$.

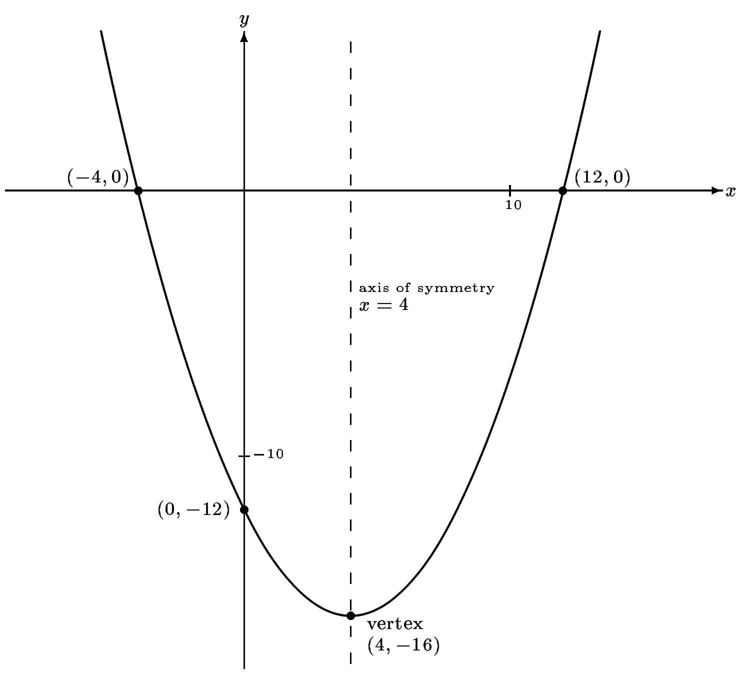
It supports the formula for f^1 because $f^{-1}(f(1)) = 1$ as the rule for inverses requires.

7. $y = \frac{1}{4}(x-4)^2 - 16.$

The steps: $y = \frac{1}{4}(x^2-8x)-12$, $y=\frac{1}{4}(x^2-8x+16-16)-12$, $y=\frac{1}{4}[(x-4)^2-16]-12$.

Be very careful to distribute to the dangling term, -16. The $\frac{1}{4}(-16)$ becomes -4.

So, the standard form is $y = \dfrac{1}{4}(x-4)^2 - 16$.



8. Originally $y = \log_2 x$.
   Now $y = \log_2(16x^2) = \log_2 16 + \log_2 x^2 = 4 + 2\log_2 x$. The y ends up 4 more than double the previous y.

9. $\log_b \frac{1}{2} = \log_b 1 - \log_b 2 = 0 - 3/4 = -3/4$.

   To evaluate $\log_2 b$, change the base to b and get $\log_2 b = \frac{\log_b b}{\log_b 2} = \frac{1}{3/4} = 4/3$.

   $b^{3/4} = 2$, so raise each side to the 4/3 power to get $b = 2^{4/3}$. It rounds off to about 2.52.

10. (a) True.
    (b) False.
    (c) True. Be careful. The denominator is the log of the square root, not the square root of the log.
    (d) False. It is not true when a is negative. But change the right side to $2\log|a| - 2$ and it is true.
    (e) True.
    (f) False.

11. Only solutions where $x > 1$ will be valid.
    $\log\left(\frac{1}{x-1}\right) = 2$.
    $10^2 = \frac{x}{x-1}$.
    $100 = \frac{x}{x-1}$.
    $100x - 100 = x$, $99x = 100$, $x = 100/99$. That's OK; it is greater than 1.

12. (a) $x = 20$ (b) $x = 500/33$ or $x = 15\frac{5}{33}$ (c) $x = 25$

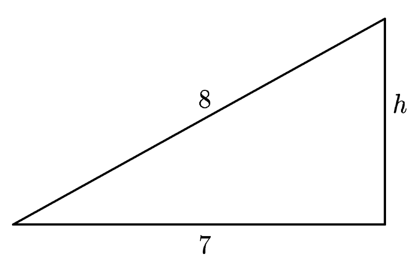
13. (a) $\dfrac{3+\sqrt{5}}{2}$ (b) 9/8 or 1.125

14. (a) 7.5 hours.
    (b) False. There were exactly 8 pounds of frut flies at time 17 hours, not at time 8 hours.
    (c) $2^{1.8} \approx 3.4822$ pounds of fruit flies.
    (d) Exactly at time 39 hours and 30 minutes.

15. (a) $\frac{3}{8}\pi$ or $67.5°$.
    (b) $157.5°$
    (c) $\frac{11\pi}{15}$.
    (d) 125 feet, because $150° = \frac{5\pi}{6}$ and $\frac{150}{\pi}\left(\frac{5\pi}{6}\right)$ feet $= 125$ feet after cancellation.

16. $\sin\theta = \dfrac{\sqrt{15}}{8}$ $\cos\theta = \dfrac{7}{8}$ $\tan\theta = \dfrac{\sqrt{15}}{7}$ $\csc\theta = \dfrac{8\sqrt{15}}{15}$ $\cot\theta = \dfrac{7\sqrt{15}}{15}$

    $\sin 2\theta = \frac{7}{32}\sqrt{15}$ $\cos 2\theta = 17/32$



    To find the third side, use $h^2 + 7^2 = 8^2$ so $h = \sqrt{64-49} = \sqrt{15}$.
    Then $\sec(90° - \theta) = 1/\cos(90° - \theta) = 1/\sin\theta = \csc\theta = \frac{8\sqrt{15}}{15}$.
    And $\csc^2\theta - 1 = \cot^2\theta = \left(\frac{7}{\sqrt{15}}\right)^2 = 49/15$.

17. $\cos 30° = \sqrt{3}/2 = 4/h$. So $h = 8/\sqrt{3} = \frac{8}{3}\sqrt{3}$ after rationalization.

18. (a) True.

$\csc x - \sin x = \frac{1-\sin^2 x}{\sin x} = \frac{\cos^2 x}{\sin x} = \left(\frac{\cos x}{\sin x}\right)\cos x.$

(b) False.

(c) False.

19. $\pi/6$ and $2\pi - \pi/6$. So $\pi/6$ and $11/\pi/6$. For the cosine, the primary solution is always the arccosine and the second one is either 2π minus the first solution or minus the first solution.

20. (a)

$$2\sin x\cos x = -\cos x$$
$$2\sin x\cos x + \cos x = 0$$
$$\cos x(2\sin x + 1) = 0$$

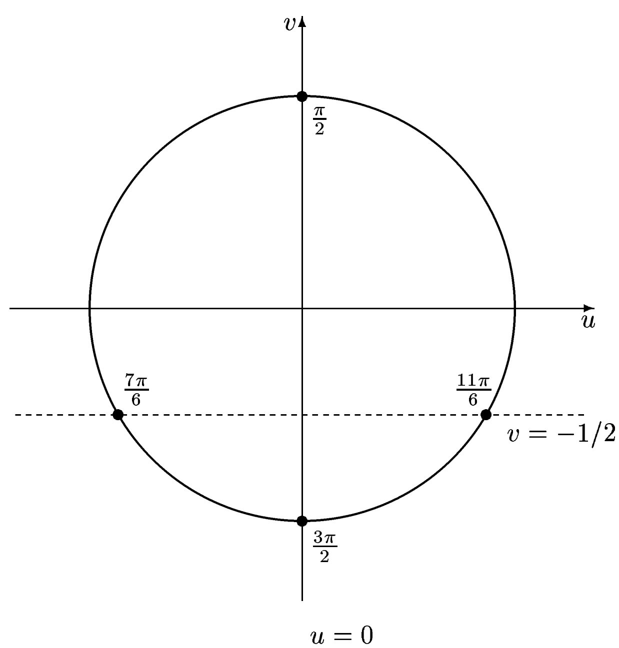
$\cos x = 0$ or $2\sin x + 1 = 0$

$x = \frac{\pi}{2}$ or $x = \frac{3\pi}{2}$

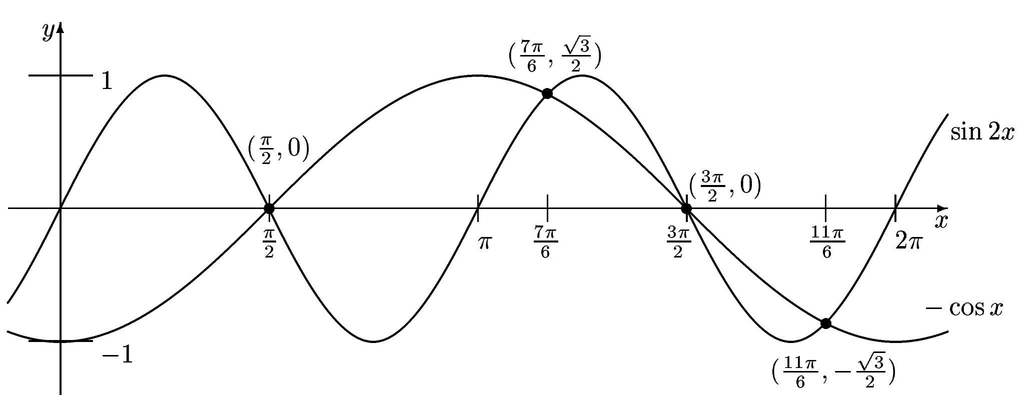
$2\sin x = -1$

$\sin x = -1/2$

$x = \frac{7\pi}{6}$ or $x = \frac{11\pi}{6}$



(b)



21. $\sin 165° = \sin 120° \cos 45° + \cos 120° \sin 45° = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \sqrt{6}/4 - \sqrt{2}/4.$

22. $\cos\theta = \dfrac{a^2+b^2-c^2}{2ab} = \dfrac{25+256-361}{160} = \dfrac{-80}{160} = -\dfrac{1}{2}.$

So $\theta = \cos^{-1}(-\frac{1}{2}) = 120°$ or $2\pi/3$.

23. $\sqrt{1-\dfrac{1}{x^2}}$ or $\dfrac{\sqrt{x^2-1}}{|x|}$

A common wrong answer is $\dfrac{\sqrt{x^2-1}}{x}$.

It is a major error to draw a right triangle with hypotenuse x and an opposite leg 1 to represent this formula. A hypotenuse must always be positive and assigning it a value of x allows that x could be negative, leading to an inadmissible situation.

A correct assignment to the hypotenuse is 1, with the opposite leg being assigned $1/x$.