More Typical Final Examination Problems

Math 130 Precalculus for the May 24, 2019 Final Exam

No books, notes, or graphing calculators; scientific calculators permitted.
Show all work with at least four-decimal-place accuracy.

1. When $f(x) = 2x^2 + 3x - 1$ and $h \neq 0$, find $\dfrac{f(2+h) - f(2)}{h}$ and simplify the result.

2. Let $f(x) = \sqrt{x}$ for all $x \geq 0$ and $g(x) = x^2$ for all real numbers.

   True or false:

   (a) $(f \circ g)(x) = x$ for all real numbers.

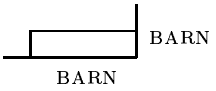
   (b) f and g are inverse functions.

3. The function f is described by the equation $f(x) = \sqrt{x+1}$ and the domain $[-1, \infty)$.

   (a) What is the range of f?
   What are the coordinates of its x and y-intercepts?

   (b) Is the function f one-to-one?
   If so, find its inverse, giving the formula and the domain.
   What are the coordinates of the x and y-intercepts of the inverse function of f?

4. Find the largest area that a farmer can enclose by constructing a rectangular pen from 26 feet of fencing, if he uses a corner of his barn for two walls of the pen.



5. For the function $f(x) = 2^x$, decide whether each of the following statements is true or false.

   (a) If you change the sign of the input, the output ends up the reciprocal of what it was previously.

   (b) If you double the input, the output ends up the square of what it was previously.

   (c) If you add 3 to the input, the output ends up 8 times as much as it was previously.

   (d) If you square the input, the new output is the old output raised to the previous input.

6. True or false:

   (a) $\dfrac{\log_a 5}{\log_a 2} = \log_2 5$.

   (b) $\dfrac{\log_a 5}{\log_a 2} = \dfrac{\log 5}{\log 2}$.

7. Decide if each statement is true or false. Then justify your answer by writing an equation.

   (a) Multiplying two numbers then taking the log gives the same result as taking each log and then adding them.

   (b) Taking the logs of two numbers then dividing those two logs gives the same result as subtracting the two numbers then taking the log of that difference.

   (c) The product of $\log_a b$ and $\log_b a$ is always equal to 1.

8. In each of parts (a) through (h), approximate the logarithm, using the properties of logarithms, given $\log_b 2 \approx 1.098$, $\log_b 3 \approx 1.740$, and $\log_b 5 \approx 2.5495$.

   (a) $\log_b 6$
   (b) $\log_b \frac{3}{5}$
   (c) $\log_b 125$
   (d) $\log_b \sqrt{3}$
   (e) $\log_b 20$
   (f) $\log_b (4b)^{-2}$
   (g) $\log_b (5b^2)$
   (h) $\log_b \sqrt[3]{2b}$

9. First decide in which intervals all valid solutions must lie.

   Then solve for x.

$$\log_2 x - \log_2(2x - 1) = -3.$$

   Check your solutions in the original equation.

10. Solve algebraically: $\log_2 x + \log_2(1-3x) = -4$.

    Hint: to solve the equation found after applying some log rules, just use the quadratic formula.

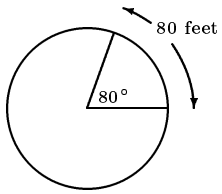
11. First decide in which intervals all valid solutions must lie.

    Then solve for x.

    $$\log_2 x + \log_2(1-2x) = -3.$$

    Check your solutions in the original equation.

12. A group has a banner that is 80 feet long. They wish to display it in the form of an arc of a circle that has angular measure of 80°. What is the radius of the circle needed for this layout? The answer may be left as $\frac{N}{\pi}$ feet.



13. A clock has a minute hand that is 4 feet long and an hour hand that is 2 1/2 feet long.

    (a) Find the angular velocity of the minute hand in radians per hour, in revolutions per hour, and in degrees per minute.

    (b) Find the linear velocity of the tip of the minute hand in inches per minute.

    (c) Find the angular velocity of the hour hand in radians per hour and in degrees per minute.

    (d) At any moment, how fast is the angle between the two hands increasing or decreasing? Give answer in degrees per minute.

    (e) In one day how far has the tip of the hour and the tip of the minute hand travelled?

    (f) How many days does it take for the tip of the minute hand to travel one mile?

14. Simplify and reduce to an expression that contains at most one trig function.

    (a) $\cos x(1+\tan x)(1-\tan x)$

    (b) $\tan x \cos^2 x$

    (c) $\cos^4 x - \sin^4 x$

    (d) $\dfrac{1+\cot^2 x}{\sin x}$

    (e) $\dfrac{\sec x}{\csc x}$

    (f) $\dfrac{\sec x}{\sin x}$

15. Find all solutions to $\sin\theta = 0.669$ in the interval $0 \le \theta < 180°$.

    Round off both answers to the nearest 0.01°.

16. Find all solutions to $\cos\theta = -0.26$ in the interval $0 \le \theta < 360°$.

    Round off both answers to the nearest 0.01°.

17. (a) Find all x between 0 and 2π for which $\sin 2x = -\sin x$.

    (b) Sketch the graphs of $\sin 2x$ and $-\sin x$ on the same axes, indicating on your sketch the points corresponding to the solutions in part (a).

18. Find the period and the amplitude of

    $$y = 5\sin\left(2x - \frac{\pi}{4}\right).$$

    Graph one period. Label with coordinates the endpoints of that period, the highest and lowest points, and all intercepts in that period.

    State the phase fraction: the portion of a period that the graph was translated right (+) or left (−).

    It might be less confusing with the 2 factored out of the expression in the parentheses.