Answers to More Typical Final Examination Problems

Math 130 Precalculus for the May 24, 2019 Final Exam

1. $\dfrac{f(2+h)-f(2)}{h} = \dfrac{(8+8h+2h^2+6+3h-1)-(2(2^2)+3(2)-1)}{h} = \dfrac{11h+2h^2}{h} = 11+2h,\ h \neq 0.$

2. (a) False: $(f \circ g)(x) = |x|$ for all real numbers.
   (b) f and g are not inverse functions. The domains and ranges do not match up properly.

3. (a) The range is $[0, \infty]$.
   The x-intercept is at $(-1, 0)$ and the y-intercept is at $(0, 1)$.
   (b) Yes, it is steadily increasing.
   To find the inverse: write $y = \sqrt{x+1}$, switch x and y to get: $x = \sqrt{y+1}$, then solve for y.
   $x^2 = y+1$, so $y = x^2 - 1$. The formula for the inverse of f is: $f^{-1}(x) = x^2 - 1$.
   However, the domain of the inverse must be the same as the range of f. The inverse is $f^{-1}(x) = x^2 - 1$ for $x \geq 0$ only.
   The y-intercept is at $(0, -1)$ and the x-intercept is at $(1, 0)$. Just switch the x- and y-coordinates of the previous points, because the inverse is the reflection across the line $y = x$.

4. Let $x =$ the width and let $z =$ the length. The area $= xz$.
   Since $x + z = 26$, we have $z = 26 - x$. Substituting $26 - x$ for z, we find that the area $= xz = x(26-x) = 26x - x^2 = -x^2 + 26x$. So for the area we have $f(x) = -x^2 + 26x$ with $a = -1 < 0$. That means that the parbola opens down and has a maximum value of $f(x)$.

$$k = c - \frac{b^2}{4a} = 0 - \frac{26^2}{4(-1)} = -\frac{576}{-4} = \frac{-576}{-4} = 169 \text{ square feet}$$

   The maximum area is equal to the maximum value of $f(x)$, which equals the maximum value of y, which is called k. Remember that the quantity to be maximized is represented by y.
   This problem could also be done by completing the square the long way.
   We find that $f(x) = -x^2 + 26x = -(x^2 - 26x) = -(x^2 - 26x + 169 - 169) = -(x-13)^2 + 169$.
   Thus $f(x) = -(x-13)^2 + 169$ and the vertex is $(13, 169) = (h, k)$.

5. All are true.

6. (a) True.
   (b) True: it is $\log_2 5$, which equals (upon change to base 10) $\dfrac{\log 5}{\log 2}$.

7. (a) True.
   $\log_a uv = \log_a u + \log_a v$
   This is the first property of logarithms.
   (b) False.
   $(\log_a u) \div (\log_a v) = \log_a(u - v)$
   $2 = \log 100 \div \log 10 \neq \log(100 - 10) = \log 90 \approx 1.95$
   (c) True.
   $(\log_a b)(\log_b a) = (\log_a b)\left(\frac{\log_a a}{\log_a b}\right) = \log_a a = 1$. The key was changing the base of $\log_b a$ to base a.

8. (a) 2.838 (b) −0.8095 (c) 7.6485 (d) 0.87 (e) 4.7455 (f) −6.392 (g) 4.5495 (h) 0.69933

9. A valid solution must greater than 1/2.

   $\log_2\left(\frac{x}{2x-1}\right) = -3.$

   $2^{-3} = \frac{x}{2x-1}.$

   $\frac{1}{8} = \frac{x}{2x-1}.$

   $8x = 2x - 1$, and $x = -\frac{1}{6}$. But this proposed solution is not greater than 1/2.

   Conclusion: there is no solution to this equation.

10. Any solution must be a positive number for which $1 - 3x > 0$ also. That means $x < 1/3$, and we have $0 < x < 1/3$. A valid solution must lie in the interval $(0, 1/3)$.

    By the product rule for logarithms: $\log_2(x - 3x^2) = -4$.
    Rewriting in exponent form: $2^{-4} = x - 3x^2$.
    Then $3x^2 - x + \frac{1}{16} = 0$.
    This could be solved by the quadratic formula, or by multiplying out by 16 then factoring, or by straight factoring using fractions.

    Quadratic formula: $\dfrac{1 \pm \sqrt{1 - 4(3)(1/16)}}{6} = \dfrac{1 \pm \frac{1}{2}}{6}.$
    The answers are $\frac{3/2}{6} = \frac{3}{12} = 1/4$ and $\frac{1/2}{6} = 1/12$.

    Multiply out: $48x^2 - 16x + 1 = 0$, factor as $(12x - 1)(4x - 1) = 0$, so $12x - 1 = 0$ and $x = 1/12$,
    or $4x - 1 = 0$ and $x = 1/4$.

    Simple factoring with fractions: $(3x - 1/4)(x - 1/4) = 0$ will give $3x - 1/4 = 0$, $x = 1/12$,
    and $x - 1/4 = 0$, $x = 1/4$.
    Check when $x = 1/4$:
    $\log_2(1/4) + \log_2(1 - 3(1/4)) = -2 + \log_2(1/4) = -2 + (-2) = -4$; it's OK.

11. Any solution must be a positive number for which $1 - 2x > 0$ also. That means $x < 1/2$, and we have $0 < x < 1/2$. A valid solution must lie in the interval $(0, 1/2)$.

    By the product rule for logarithms: $\log_2(x - 2x^2) = -3$.
    Rewriting in exponent form: $2^{-3} = x - 2x^2$.
    Then $2x^2 - x + \frac{1}{8} = 0$.
    This could be solved by the quadratic formula, or by multiplying out by 8 then factoring, or by straight factoring using fractions.

    Quadratic formula: $\dfrac{1 \pm \sqrt{1 - 4(2)(1/8)}}{4} = \dfrac{1 \pm 0}{4} = 1/4.$

    Multiply out: $16x^2 - 8x + 1 = 0$, perfect square $(4x - 1)^2 = 0$, so $4x - 1 = 0$ and $x = 1/4$.

    Simple factoring with fractions: $(2x - 1/2)(x - 1/4) = 0$ will give $2x - 1/2 = 0$, $x = 1/4$,
    and $x - 1/4 = 0$, $x = 1/4$.

    Check:
    $\log_2(1/4) + \log_2(1 - 2(1/4)) = -2 + \log_2(1/2) = -2 + (-1) = -3$; it's OK.

12. Convert 80° to radians: $80^\circ \times \frac{\pi \text{ radians}}{180^\circ} = \frac{4\pi}{9}$.

    80 feet $= \frac{4\pi}{9} \times r$. Since $\frac{4\pi}{9}$ is a unitless ratio, do not include the word radians in this equation.

    $r = \frac{180}{\pi}$ feet, after solving.

13. (a) It is 2π radians per hour, as it goes around once each hour. That is one revolution per hour. For the degrees, 360 degrees per hour works out to 6 degrees per minute.

    (b) It is 2π radians per hour, thus $\pi/30$ radians per minute.

    4 feet is 48 inches.
    $s = r\theta =$ (48 inches) $\times\, \pi/30$ radians per minute, giving 1.6π inches per minute.

    (c) It takes 12 hours for the hour hand to complete one revolution.
    In one hour, the hour hand makes one-twelfth of a revolution. Its rotational velocity is therefore $\frac{2\pi}{12}$ radians per hour. That is 30 degrees per hour, which works out to 1/2 degree per minute.

    (d) Subtract the angular velocities to get $5\frac{1}{2}$ degrees per minute.

    (e) The tip of the hour hand has travelled 10π feet in 24 hours, while the tip of the minute hand has travelled 192π feet in 24 hours.

    (f) 8.75352187 days, which is 8 days, 18 hours, 5 minutes, and about 4.3 seconds.

14. (a) $\cos x(1 + \tan x)(1 - \tan x) = \cos x(1 - \tan^2 x) = \cos x(1 - (\sec^2 x - 1)) = \cos x(2 - \sec^2 x) =$

    $$2\cos x - \frac{1}{\cos x} = \frac{2\cos^2 x - 1}{\cos x}.$$ This answer could also be given as $\frac{\cos 2x}{\cos x}$.

    (b) $\tan x \cos^2 x = (\sin x / \cos x)\cos^2 x = \sin x \cos x = \frac{1}{2}\sin 2x$.

    (c) $\cos^4 x - \sin^4 x = (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x) = (1)(\cos 2x) = \cos 2x$

    (d) $\frac{1+\cot^2 x}{\sin x} = \frac{\csc^2 x}{\sin x} = \csc^2 x \left(\frac{1}{\sin x}\right) = \csc^2 x(\csc x) = \csc^3 x$.

    (e) $\frac{\sec x}{\csc x} = \left(\frac{1}{\cos x}\right)\left(\frac{1}{\csc x}\right) = \left(\frac{1}{\cos x}\right)\sin x = \frac{\sin x}{\cos x} = \tan x$.

    (f) $\frac{\sec x}{\sin x} = \sec x\left(\frac{1}{\sin x}\right) = \left(\frac{1}{\cos x}\right)\left(\frac{1}{\sin x}\right) = \frac{1}{\sin x \cos x} = \frac{1}{(1/2)\sin 2x} = 2\csc 2x$.

15. 41.99° and $180^\circ - 41.99^\circ = 138.01^\circ$. For the sine, the first solution is obtained from the arcsine and a second solution is 180° minus the first.

16. 105.07° and $360^\circ - 105.07^\circ = 254.93^\circ$.

17. (a)

$$2\sin x\cos x = -\sin x$$
$$2\sin x\cos x + \sin x = 0$$
$$\sin x(2\cos x + 1) = 0$$

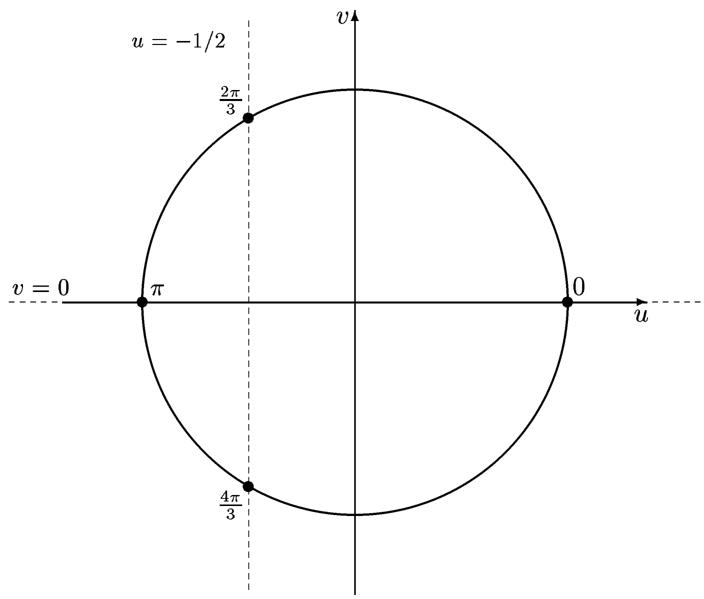
$\sin x = 0$ or $2\cos x + 1 = 0$

$x = 0$ or $x = \pi$ or $x = 2\pi$

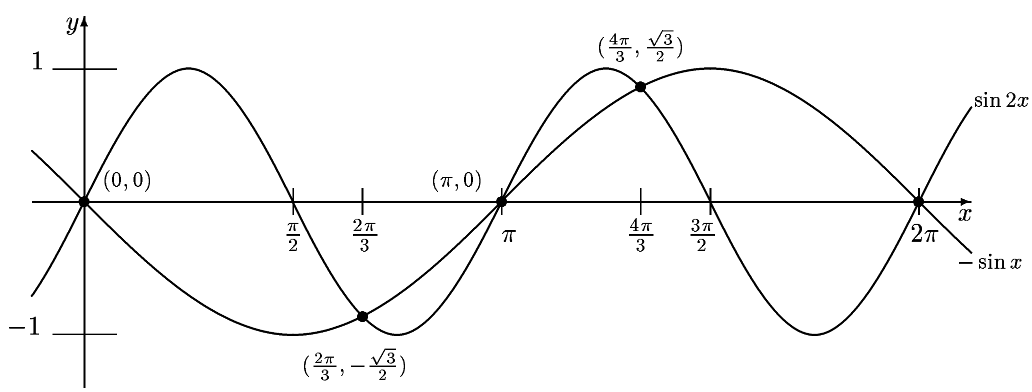
$2\cos x = -1$

$\cos x = -1/2$

$x = \frac{2\pi}{3}$ or $x = \frac{4\pi}{3}$



(b)



18.

The period is $\dfrac{2\pi}{|a|} = \dfrac{2\pi}{2} = \pi$.

The amplitude is 5.

Factor as $5\sin\left[2\left(x - \frac{\pi}{8}\right)\right]$.

So shift the graph of $y = 5\sin 2x$ $\pi/8$ to the right.