Sample Final Examination

Math 130 Precalculus for the December 21, 2018 Final Exam

No books, notes, or graphing calculators; scientific calculators permitted.
Show all work with at least four-decimal-place accuracy.

1. When $f(x) = 2x^2 + 3x - 1$ and $h \neq 0$, find $\dfrac{f(2+h) - f(2)}{h}$ and simplify the result.

2. In each case decide whether the function with the given rule is even, odd, or neither. Explain your reasoning or support your answer.

   (a) $f(x) = \dfrac{1}{x-1} + \dfrac{1}{x} + \dfrac{1}{x+1}$

   (b) $g(x) = |x+1|$

   (c) $h(x) = |x+3| - |x-3|$

   (d) $i(x) = \sqrt{(4-x)(4+x)}$

   (e) $j(x) = \sqrt{x+1} \cdot \sqrt{x-1}$, defined for real-valued outputs only.

3. In each part, how is the graph of the given function related to the graph of the parent function $f(x) = \sqrt{x}$?

   (a) $g(x) = \sqrt{x+5} + 5$.

   (b) $h(x) = 2\sqrt{x-3}$.

   (c) $i(x) = -\sqrt{-x-3}$.

4. Determine whether $f(x) = \dfrac{4}{-5x+3}$ has an inverse function. If it does, find the inverse function.

   If it has an inverse function:

   What is the value of $f(1)$? Call it a.

   Apply the function f inverse (written as f^{-1}) to a. Is the result OK?

5. Complete the square, getting the equation $y = \frac{1}{4}x^2 - 2x - 12$ into standard form, and sketch its graph.

   Plot the vertex, axis of symmetry, and all intercepts, labelling with both coordinates or the equation.

6. When the price of a certain DVD is \$20, one million are sold. For each \$1 decrease in price, the number sold increases by 100,000.

   (a) Write the revenue R from the DVD sales as a function of the price x of the DVD.

   (b) What price will yield maximum revenue?
   What is the maximum revenue?

7. At \$10 per ticket, Willie Williams and the Wranglers will fill all 8000 seats in the Assembly Center. The manager knows that for every \$1 increase in the price, 500 tickets will go unsold.

   (a) Write the number of tickets sold n as a function of ticket price p.

   (b) Write the total revenue as a function of the ticket price.

   (c) What ticket price will maximize the revenue?

8. Decide if each statement is true or false. Then justify your answer by writing an equation.

   (a) Multiplying two numbers then taking the log gives the same result as taking each log and then adding them.

   (b) Taking the logs of two numbers then dividing those two logs gives the same result as subtracting the two numbers then taking the log of that difference.

   (c) The product of $\log_a b$ and $\log_b a$ is always equal to 1.

9. In each of parts (a) through (h), approximate the logarithm, using the properties of logarithms, given $\log_b 2 \approx 1.098$, $\log_b 3 \approx 1.740$, and $\log_b 5 \approx 2.5495$.

   (a) $\log_b 6$

   (b) $\log_b \frac{3}{5}$

   (c) $\log_b 125$

   (d) $\log_b \sqrt{3}$

   (e) $\log_b 20$

   (f) $\log_b (4b)^{-2}$

   (g) $\log_b (5b^2)$

   (h) $\log_b \sqrt[3]{2b}$

10. Solve algebraically:

    (a) $\log x + \log(x - 15) = 2$.

    (b) $\log x - \log(x - 15) = 2$.

    (c) $\log 24x - \log(1 + \sqrt{x}) = 2$.

11. A population of fruit flies is increasing according to the law of exponential growth. At time 2 hours there are 2 pounds of flies and at time 32 hours there are 32 pounds of flies.

    (a) Find the exact value of the doubling time. (No calculator is necessary.)

    (b) True or false: at time 8 hours there were exactly 8 pounds of fruit flies.

    (c) If false, about how many pounds of fruit flies were there at time 8 hours (to the nearest three-decimal accuracy or as an exact radical expression).

    (d) At exactly what time will there be 64 pounds of fruit flies?

12. (a) Rewrite in radian measure as a fractional multiple of π and in degree measure: $3/16$ of a revolution.

    (b) Rewrite in degree measure: $\dfrac{7\pi}{8}$.

    (c) Rewrite in radian measure as a fractional multiple of π in lowest terms: $132°$.

    (d) Find the length of the arc on a circle of radius $150/\pi$ feet intercepted by a central angle of $150°$.

13. A clock has a minute hand that is 4 feet long and an hour hand that is 2 1/2 feet long.

    (a) Find the angular velocity of the minute hand in radians per hour, in revolutions per hour, and in degrees per minute.

    (b) Find the linear velocity of the tip of the minute hand in inches per minute.

    (c) Find the angular velocity of the hour hand in radians per hour and in degrees per minute.

    (d) At any moment, how fast is the angle between the two hands increasing or decreasing? Give answer in degrees per minute.

    (e) In one day how far has the tip of the hour and the tip of the minute hand travelled?

    (f) How many days does it take for the tip of the minute hand to travel one mile?

14. Find all solutions to $\cos\theta = \sqrt{3}/2$ in the interval $0 \le \theta < 2\pi$.

15. A right triangle has an acute angle θ with $\sec\theta = \frac{8}{7}$. Find the exact values of the other five trigonometric functions of θ, in fractional form. Some of the expressions will involve square roots; do not convert the square roots to decimals.

    Then find the exact values of $\sec(90° - \theta)$ and of $\csc^2\theta - 1$, also in fractional form.

    Hint. First sketch a right triangle corresponding to that secant. Next use the Pythagorean Theorem to determine the third side. Then find the other five trigonometric functions of θ.

    For the other two values, use the appropriate trigonometric identities.

16. Simplify and reduce to an expression that contains at most one trig function.

    (a) $\cos x(1 + \tan x)(1 - \tan x)$

    (b) $\tan x \cos^2 x$

    (c) $\cos^4 x - \sin^4 x$

    (d) $\dfrac{1 + \cot^2 x}{\sin x}$

    (e) $\dfrac{\sec x}{\csc x}$

    (f) $\dfrac{\sec x}{\sin x}$

17. In each part, decide whether the identity is true or false. If true, verify it.

    (a) $\csc x - \sin x = \cot x \cos x$.

    (b) $\dfrac{1}{\cos x} - \dfrac{1}{\sec x} = \cos x - \sec x$.

    (c) $(1 + \cot x)^2 = \csc^2 x$.

18. (a) Find all solutions with $0 \le x \le 2\pi$ for $\sin 2x = -\cos x$.

    (b) Graph $\sin 2x$ and $-\cos x$ on the same axes and indicate on your sketch the points corresponding to the solutions in part (a).

19. Find the exact value of $\sin 165°$.

20. Find the size of the angle between the sides of lengths 5 and 16 in a triangle with sides of 5, 16, and 19.