Answers to the Sample Final Examination

Math 130 Precalculus for the December 21, 2018 Final Exam

1. $\dfrac{f(2+h)-f(2)}{h}=\dfrac{(8+8h+2h^2+6+3h-1)-(2(2^2)+3(2)-1)}{h}=\dfrac{11h+2h^2}{h}=11+2h,\ h\neq 0.$

2. (a) f is odd because it simplifies to $\frac{3x^2-1}{x(x^2-1)}$, and when x has opposite sign, so does this fraction.
   (b) g is neither even nor odd because $g(3)=4$ while $g(-3)=2$.
   (c) h is odd because $h(a)=|a+3|-|a-3|$ and $h(-a)=|-a+3|-|-a-3|=|a-3|-|a+3|=-h(a)$. For support, show that $h(4)=6$ and $h(-4)=-6$.
   (d) i is even because $i(x)=\sqrt{16-x^2}$ showing that it is even by the rule of even powers and also that the domain is balanced, being $[-4,4]$.
   (e) j is neither even nor odd because the domain, $x\geq 1$, is not balanced.

3. (a) The graph of $y=\sqrt{x}$ was moved 5 units to the left and 5 units up.
   (b) The graph of $y=\sqrt{x}$ was stretched vertically by a factor of 2 and moved 3 units to the right.
   (c) It is best to rewrite this as $i(x)=-\sqrt{-(x+3)}$.
   Then graph an intermediate function, $y=\sqrt{-x}$, first. Its graph is the graph of $f(x)$ reflected across the y-axis.
   Then replace x with $(x+3)$ to get $y=\sqrt{-(x+3)}$. This moves the previous graph 3 units to the left.
   Finally, $i(x)=-\sqrt{-x-3}$ is just the previous graph reflected across the x-axis, since all y values will assume opposite sign.

   Answer: The graph of $y=\sqrt{x}$ was reflected across the y-axis, then shifted 3 units to the left, and then reflected across the x-axis.

   Verify by noting that the point $(9,3)$ on f will end up as the point $(-12,-3)$ on i and that the point $(-12,-3)$ is indeed a solution of the equation for i because $-3=-\sqrt{-(-12)-3}$.

4. $f^{-1}(x) = \dfrac{3x-4}{5x}$. It was found by solving $x = \dfrac{4}{-5y+3}$ for y. It is a function.

As a verbal string it is divide by 4; take reciprocal; subtract 3; change sign; divide by 5. This is the formula

$$-\frac{1}{5}\left(\frac{4}{x}-3\right).$$

The two answers, while different formulas, are algebraically equivalent.

The orginal verbal string was (before inversion) multiply by 5, change sign, add 3, take reciprocal, multiply by 4. All that had to be done was to find the inverse of each operation and apply them in the opposite order as they were in the original function.

Because this problem is not easily conceptualized, one should apply the two results to a number of his choosing to see if they work.
Let $x = 3$, so $f(3) = \frac{4}{-15+3} = \frac{4}{-12} = -\frac{1}{3}$.

Then $f^{-1}(-1/3) = \frac{3(-1/3)-4}{5(-1/3)} = \frac{-5}{-5/3} = 3$. It checks.

And the verbal: $-1/3$ divided by 4 is $-1/12$, reciprocal is -12, subtract 3 gives -15, change sign gives 15, divide by 5 gives 3, as expected.

$f(1) = -2$, so $a = -2$.
$f^{-1}(-2) = \frac{3(-2)-4}{5(-2)} = \frac{-10}{-10} = 1$.

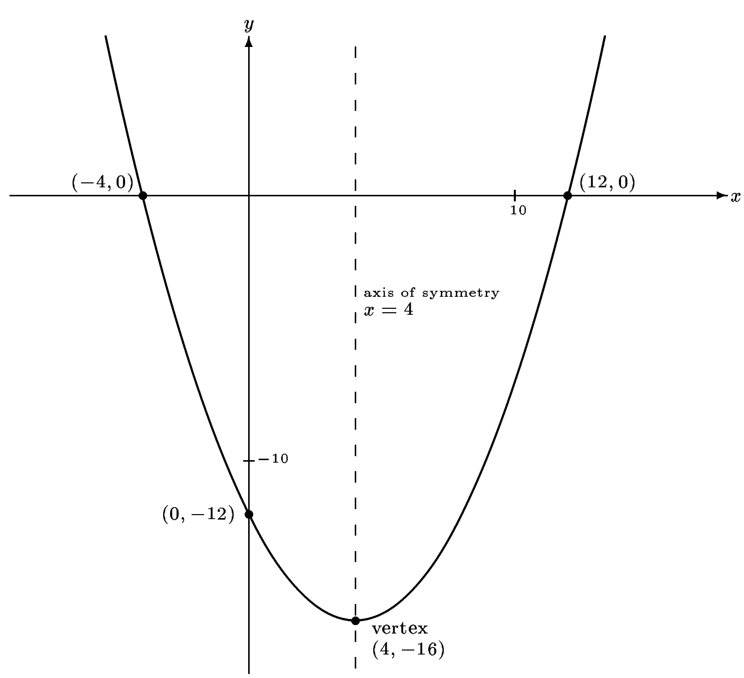
It supports the formula for f^1 because $f^{-1}(f(1)) = 1$ as the rule for inverses requires.

5. $y = \frac{1}{4}(x-4)^2 - 16$.

The steps: $y = \frac{1}{4}(x^2-8x) - 12$, $y = \frac{1}{4}(x^2-8x+16-16) - 12$, $y = \frac{1}{4}[(x-4)^2-16] - 12$.

Be very careful to distribute to the dangling term, -16. The $\frac{1}{4}(-16)$ becomes -4.

So, the standard form is $y = \dfrac{1}{4}(x-4)^2 - 16$.



6. (a) The number sold $= 1,000,000 + 100,000(20 - x) = 3{,}000{,}000 - 100,000x$; the revenue is x times that, which equals $-100{,}000x^2 + 3{,}000{,}000x$.
   (b) The maximum revenue is for $x = -b/2a = \$15$. The maximum revenue is 22.5 million dollars.

7. (a) Suppose p is the ticket price. Then i, the increase is equal to $p - 10$.
   The number of tickets sold is 8000 minus 500 times the increase i.
   $n = 8000 - 500i$.
   Substitute $p - 10$ for i to get $n = 8000 - 500(p - 10) = 8000 - 500p + 5000 = -500p + 13{,}000$.
   (b) $R = -500p^2 + 13{,}000p$.
   (c) $R = -500(p^2 - 26p) = -500(p^2 - 26p + 169 - 169) = -500(p - 13)^2 + 84{,}500$.
   This parabola opens down and is maximized at the vertex, when $p = 13$, meaning a price of \$13.

8. (a) True.
   $\log_a uv = \log_a u + \log_a v$
   This is the first property of logarithms.
   (b) False.
   $(\log_a u) \div (\log_a v) = \log_a(u - v)$
   $2 = \log 100 \div \log 10 \neq \log(100 - 10) = \log 90 \approx 1.95$
   (c) True.
   $(\log_a b)(\log_b a) = (\log_a b)\left(\frac{\log_a a}{\log_a b}\right) = \log_a a = 1$. The key was changing the base of $\log_b a$ to base a.

9. (a) 2.838 (b) -0.8095 (c) 7.6485 (d) 0.87 (e) 4.7455 (f) -6.392 (g) 4.5495 (h) 0.69933

10. (a) $x = 20$ (b) $x = 500/33$ or $x = 15\frac{5}{33}$ (c) $x = 25$

11. (a) 7.5 hours.
    (b) False. There were exactly 8 pounds of frut flies at time 17 hours, not at time 8 hours.
    (c) $2^{1.8} \approx 3.4822$ pounds of fruit flies.
    (d) Exactly at time 39 hours and 30 minutes.

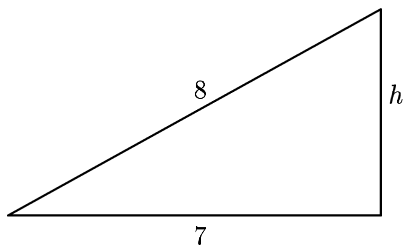
12. (a) $\frac{3}{8}\pi$ or $67.5°$.
    (b) $157.5°$
    (c) $\frac{11\pi}{15}$.
    (d) 125 feet, because $150° = \frac{5\pi}{6}$ and $\frac{150}{\pi}\left(\frac{5\pi}{6}\right)$ feet $= 125$ feet after cancellation.

13. (a) It is 2π radians per hour, as it goes around once each hour. That is one revolution per hour.
    For the degrees, 360 degrees per hour works out to 6 degrees per minute.
    (b) It is 2π radians per hour, thus $\pi/30$ radians per minute.
    4 feet is 48 inches.
    $s = r\theta =$ (48 inches) $\times\ \pi/30$ radians per minute, giving 1.6π inches per minute.
    (c) It takes 12 hours for the hour hand to complete one revolution.
    In one hour, the hour hand makes one-twelfth of a revolution. Its rotational velocity is therefore $\frac{2\pi}{12}$ radians per hour. That is 30 degrees per hour, which works out to 1/2 degree per minute.
    (d) Subtract the angular velocities to get $5\frac{1}{2}$ degrees per minute.
    (e) The tip of the hour hand has travelled 10π feet in 24 hours, while the tip of the minute hand has travelled 192π feet in 24 hours.
    (f) 8.75352187 days, which is 8 days, 18 hours, 5 minutes, and about 4.3 seconds.

14. $\pi/6$ and $2\pi - \pi/6$. So $\pi/6$ and $11/\pi/6$. For the cosine, the primary solution is always the arccosine and the second one is either 2π minus the first solution or minus the first solution.

15. $\sin\theta = \dfrac{\sqrt{15}}{8}$ $\cos\theta = \dfrac{7}{8}$ $\tan\theta = \dfrac{\sqrt{15}}{7}$ $\csc\theta = \dfrac{8\sqrt{15}}{15}$ $\cot\theta = \dfrac{7\sqrt{15}}{15}$

$\sin 2\theta = \frac{7}{32}\sqrt{15}$ $\cos 2\theta = 17/32$



To find the third side, use $h^2 + 7^2 = 8^2$ so $h = \sqrt{64-49} = \sqrt{15}$.

Then $\sec(90° - \theta) = 1/\cos(90° - \theta) = 1/\sin\theta = \csc\theta = \frac{8\sqrt{15}}{15}$.

And $\csc^2\theta - 1 = \cot^2\theta = \left(\frac{7}{\sqrt{15}}\right)^2 = 49/15$.

16. (a) $\cos x(1+\tan x)(1-\tan x) = \cos x(1-\tan^2 x) = \cos x(1-(\sec^2 x - 1)) = \cos x(2-\sec^2 x) =$

$$2\cos x - \frac{1}{\cos x} = \frac{2\cos^2 x - 1}{\cos x}.$$ This answer could also be given as $\dfrac{\cos 2x}{\cos x}$.

(b) $\tan x \cos^2 x = (\sin x/\cos x)\cos^2 x = \sin x \cos x = \frac{1}{2}\sin 2x$.

(c) $\cos^4 x - \sin^4 x = (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x) = (1)(\cos 2x) = \cos 2x$

(d) $\frac{1+\cot^2 x}{\sin x} = \frac{\csc^2 x}{\sin x} = \csc^2 x\left(\frac{1}{\sin x}\right) = \csc^2 x(\csc x) = \csc^3 x$.

(e) $\frac{\sec x}{\csc x} = \left(\frac{1}{\cos x}\right)\left(\frac{1}{\csc x}\right) = \left(\frac{1}{\cos x}\right)\sin x = \frac{\sin x}{\cos x} = \tan x$.

(f) $\frac{\sec x}{\sin x} = \sec x\left(\frac{1}{\sin x}\right) = \left(\frac{1}{\cos x}\right)\left(\frac{1}{\sin x}\right) = \frac{1}{\sin x\cos x} = \frac{1}{(1/2)\sin 2x} = 2\csc 2x$.

17. (a) True.

$\csc x - \sin x = \frac{1-\sin^2 x}{\sin x} = \frac{\cos^2 x}{\sin x} = \left(\frac{\cos x}{\sin x}\right)\cos x$.

(b) False.

(c) False.

18. (a)

$$\begin{aligned} 2\sin x\cos x &= -\cos x \\ 2\sin x\cos x + \cos x &= 0 \\ \cos x(2\sin x + 1) &= 0 \end{aligned}$$

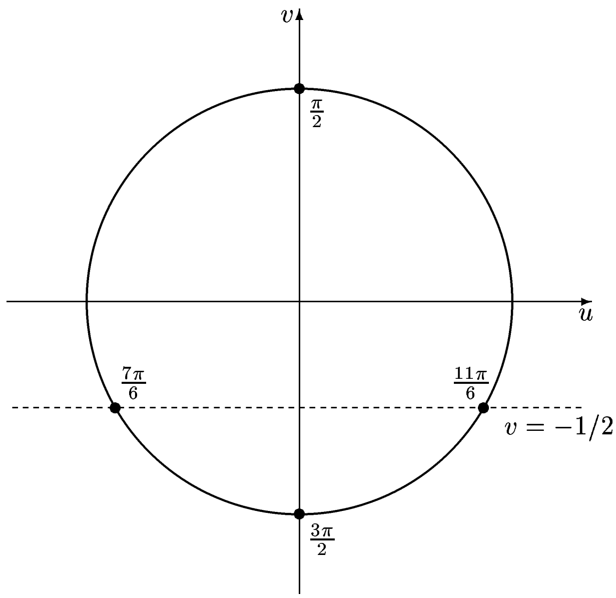
$\cos x = 0$ or $2\sin x + 1 = 0$

$x = \frac{\pi}{2}$ or $x = \frac{3\pi}{2}$

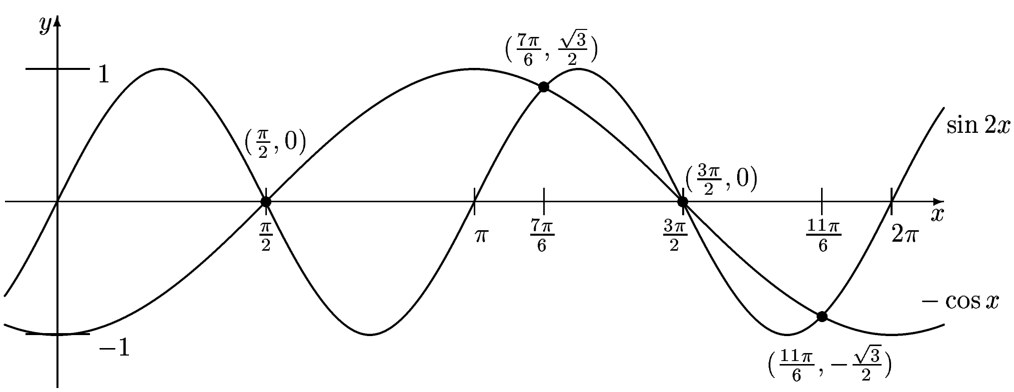
$2\sin x = -1$

$\sin x = -1/2$

$x = \frac{7\pi}{6}$ or $x = \frac{11\pi}{6}$



(b)



19. $\sin 165° = \sin 120° \cos 45° + \cos 120° \sin 45° = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \sqrt{6}/4 - \sqrt{2}/4.$

20. $\cos\theta = \dfrac{a^2+b^2-c^2}{2ab} = \dfrac{25+256-361}{160} = \dfrac{-80}{160} = -\dfrac{1}{2}.$

So $\theta = \cos^{-1}(-\frac{1}{2}) = 120°$ or $2\pi/3$.