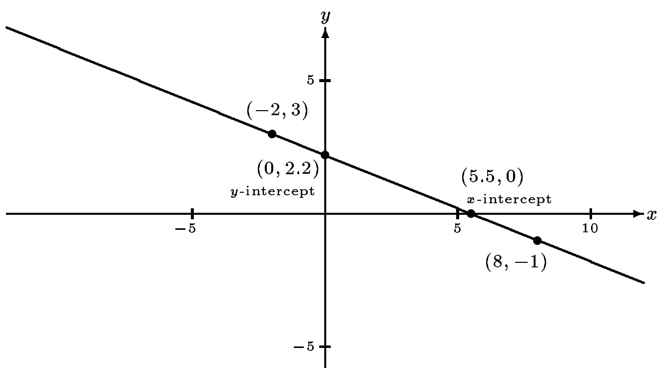
Answers to More Typical Final Examination Problems

Math 130 Precalculus for the December 21, 2018 Final Exam

1. $$y = -\frac{2}{5}x + \frac{11}{5} \quad \text{or} \quad y = -0.4x + 2.2$$

Also $$y - 3 = -\frac{2}{5}(x+2) \quad \text{or} \quad y + 1 = -\frac{2}{5}(x-8)$$

Also $$2x + 5y = 11.$$



Work: $m = \frac{-1-3}{8-(-2)} = \frac{-4}{10} = -2/5$ and $y - 3 = -\frac{2}{5}(x+2)$.

2. (a) False: $(f \circ g)(x) = |x|$ for all real numbers.
   (b) f and g are not inverse functions. The domains and ranges do not match up properly.

3. (a) The range is $[0, \infty]$.

   The x-intercept is at $(-1, 0)$ and the y-intercept is at $(0, 1)$.

   (b) Yes, it is steadily increasing.

   To find the inverse: write $y = \sqrt{x+1}$, switch x and y to get: $x = \sqrt{y+1}$, then solve for y.

   $x^2 = y + 1$, so $y = x^2 - 1$. The formula for the inverse of f is: $f^{-1}(x) = x^2 - 1$.

   However, the domain of the inverse must be the same as the range of f. The inverse is $f^{-1}(x) = x^2 - 1$ for $x \geq 0$ only.

   The y-intercept is at $(0, -1)$ and the x-intercept is at $(1, 0)$. Just switch the x- and y-coordinates of the previous points, because the inverse is the reflection across the line $y = x$.

4. Let $x =$ the width and let $z =$ the length. The area $= xz$.
Since $x + z = 26$, we have $z = 26 - x$. Substituting $26 - x$ for z, we find that the area $= xz = x(26 - x) = 26x - x^2 = -x^2 + 26x$. So for the area we have $f(x) = -x^2 + 26x$ with $a = -1 < 0$. That means that the parbola opens down and has a maximum value of $f(x)$.

$$k = c - \frac{b^2}{4a} = 0 - \frac{26^2}{4(-1)} = -\frac{576}{-4} = \frac{-576}{-4} = 169 \text{ square feet}$$

The maximum area is equal to the maximum value of $f(x)$, which equals the maximum value of y, which is called k. Remember that the quantity to be maximized is represented by y.
This problem could also be done by completing the square the long way.
We find that $f(x) = -x^2 + 26x = -(x^2 - 26x) = -(x^2 - 26x + 169 - 169) = -(x - 13)^2 + 169$.
Thus $f(x) = -(x - 13)^2 + 169$ and the vertex is $(13, 169) = (h, k)$.

5. (a) $p = 50 - n$.
   (b) $R = 50n - n^2$.
   (c) $R = -(n-25)^2 + 625$, so the maximum revenue is \$625.

6. Originally $y = \log_2 x$.
   Now $y = \log_2(32x) = \log_2 32 + \log_2 x = 5 + \log_2 x$. The y ends up 5 larger, so the change in y is 5.

7. $\log_b \frac{1}{2} = \log_b 1 - \log_b 2 = 0 - 3/4 = -3/4$.

   To evaluate $\log_2 b$, change the base to b and get $\log_2 b = \frac{\log_b b}{\log_b 2} = \frac{1}{3/4} = 4/3$.

   $b^{3/4} = 2$, so raise each side to the 4/3 power to get $b = 2^{4/3}$. It rounds off to about 2.52.

8. (a) True.
   (b) False.
   (c) True. Be careful. The denominator is the log of the square root, not the square root of the log.
   (d) False. It is not true when a is negative. But change the right side to $2\log|a| - 2$ and it is true.
   (e) True.
   (f) False.

9. (a) True.
   (b) True: it is $\log_2 5$, which equals (upon change to base 10) $\dfrac{\log 5}{\log 2}$.

10. Only solutions where $x > 1$ will be valid.

    $\log\left(\frac{1}{x-1}\right) = 2$.
    $10^2 = \frac{x}{x-1}$.
    $100 = \frac{x}{x-1}$.
    $100x - 100 = x$, $99x = 100$, $x = 100/99$. That's OK; it is greater than 1.

11. A valid solution must greater than 1/2.

    $\log_2\left(\frac{x}{2x-1}\right) = -3$.
    $2^{-3} = \frac{x}{2x-1}$.
    $\frac{1}{8} = \frac{x}{2x-1}$.
    $8x = 2x - 1$, and $x = -\frac{1}{6}$. But this proposed solution is not greater than 1/2.

    Conclusion: there is no solution to this equation.

12. Any solution must be a positive number for which $1 - 3x > 0$ also. That means $x < 1/3$, and we have $0 < x < 1/3$. A valid solution must lie in the interval $(0, 1/3)$.

    By the product rule for logarithms: $\log_2(x - 3x^2) = -4$.

    Rewriting in exponent form: $2^{-4} = x - 3x^2$.

    Then $3x^2 - x + \frac{1}{16} = 0$.

    This could be solved by the quadratic formula, or by multiplying out by 16 then factoring, or by straight factoring using fractions.

    Quadratic formula: $\dfrac{1 \pm \sqrt{1 - 4(3)(1/16)}}{6} = \dfrac{1 \pm \frac{1}{2}}{6}$.

    The answers are $\frac{3/2}{6} = \frac{3}{12} = 1/4$ and $\frac{1/2}{6} = 1/12$.

    Multiply out: $48x^2 - 16x + 1 = 0$, factor as $(12x - 1)(4x - 1) = 0$, so $12x - 1 = 0$ and $x = 1/12$,
    or $4x - 1 = 0$ and $x = 1/4$.

    Simple factoring with fractions: $(3x - 1/4)(x - 1/4) = 0$ will give $3x - 1/4 = 0$, $x = 1/12$,
    and $x - 1/4 = 0$, $x = 1/4$.

    Check when $x = 1/4$:

    $\log_2(1/4) + \log_2(1 - 3(1/4)) = -2 + \log_2(1/4) = -2 + (-2) = -4$; it's OK.

13. Any solution must be a positive number for which $1 - 2x > 0$ also. That means $x < 1/2$, and we have $0 < x < 1/2$. A valid solution must lie in the interval $(0, 1/2)$.

    By the product rule for logarithms: $\log_2(x - 2x^2) = -3$.

    Rewriting in exponent form: $2^{-3} = x - 2x^2$.

    Then $2x^2 - x + \frac{1}{8} = 0$.

    This could be solved by the quadratic formula, or by multiplying out by 8 then factoring, or by straight factoring using fractions.

    Quadratic formula: $\dfrac{1 \pm \sqrt{1 - 4(2)(1/8)}}{4} = \dfrac{1 \pm 0}{4} = 1/4$.

    Multiply out: $16x^2 - 8x + 1 = 0$, perfect square $(4x - 1)^2 = 0$, so $4x - 1 = 0$ and $x = 1/4$.

    Simple factoring with fractions: $(2x - 1/2)(x - 1/4) = 0$ will give $2x - 1/2 = 0$, $x = 1/4$,
    and $x - 1/4 = 0$, $x = 1/4$.

    Check:

    $\log_2(1/4) + \log_2(1 - 2(1/4)) = -2 + \log_2(1/2) = -2 + (-1) = -3$; it's OK.

14. Convert 80° to radians: $80° \times \frac{\pi \text{ radians}}{180°} = \frac{4\pi}{9}$.

    80 feet $= \frac{4\pi}{9} \times r$. Since $\frac{4\pi}{9}$ is a unitless ratio, do not include the word radians in this equation.

    $r = \frac{180}{\pi}$ feet, after solving.

15. $\cos 30° = \sqrt{3}/2 = 4/h$. So $h = 8/\sqrt{3} = \frac{8}{3}\sqrt{3}$ after rationalization.

16. 41.99° and 180° − 41.99° = 138.01°. For the sine, the first solution is obtained from the arcsine and a second solution is 180° minus the first.

17. 105.07° and 360° − 105.07° = 254.93°.

18. (a)

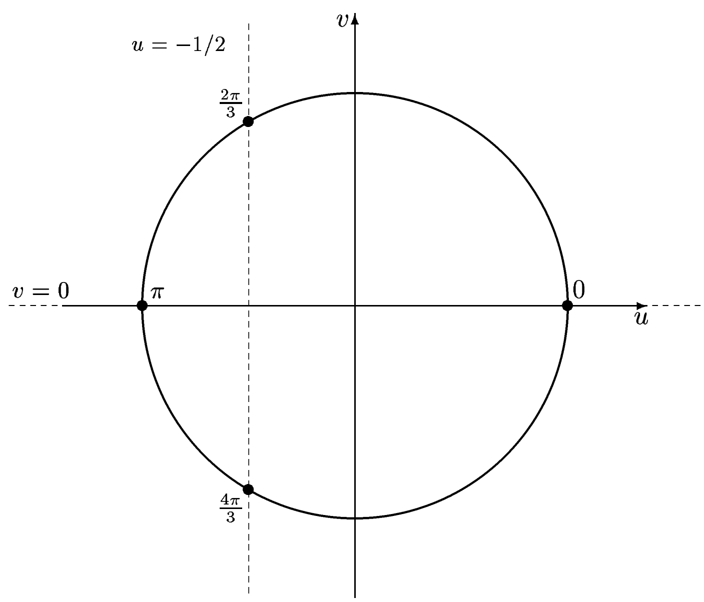
$$2\sin x\cos x = -\sin x$$
$$2\sin x\cos x + \sin x = 0$$
$$\sin x(2\cos x + 1) = 0$$

$\sin x = 0$ or $2\cos x + 1 = 0$

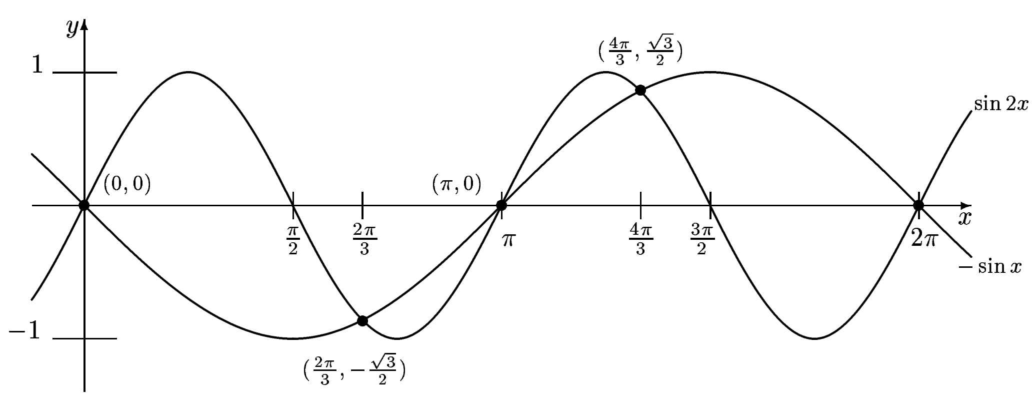
$x = 0$ or $x = \pi$ or $x = 2\pi$ $\qquad$ $2\cos x = -1$

$\cos x = -1/2$

$x = \frac{2\pi}{3}$ or $x = \frac{4\pi}{3}$



(b)



19.

The period is $\dfrac{2\pi}{|a|} = \dfrac{2\pi}{2} = \pi$.

The amplitude is 5.

Factor as $\quad 5\sin\left[2\left(x - \frac{\pi}{8}\right)\right]$.

So shift the graph of $y = 5\sin 2x$ $\pi/8$ to the right.