

Sample of Typical Final Examination Problems

Math 130 Precalculus for the May 18, 2018 Final Exam

No books, notes, or graphing calculators; scientific calculators permitted.

Show all work with at least four-decimal-place accuracy.

1. Find an equation of the line passing through the points $(8, -1)$ and $(-2, 3)$. Sketch the line and label both intercepts with their coordinates.
2. Write the standard form of the equation of a circle with endpoints of a diameter $(5, 6)$ and $(17, 15)$. State the coordinates of the center and the highest point, and find the length of the radius.
3. When $f(x) = 2x^2 + 3x - 1$ and $h \neq 0$, find $\frac{f(2+h) - f(2)}{h}$ and simplify the result.
4. In each case decide whether the function with the given rule is even, odd, or neither. Explain your reasoning or support your answer.

(a) $f(x) = \frac{1}{x-1} + \frac{1}{x} + \frac{1}{x+1}$

(b) $g(x) = |x+1|$

(c) $h(x) = |x+3| - |x-3|$

(d) $i(x) = \sqrt{(4-x)(4+x)}$

(e) $j(x) = \sqrt{x+1}\sqrt{x-1}$, defined for real-valued outputs only.

5. Let $f(x) = \sqrt{x}$ for all $x \geq 0$ and $g(x) = x^2$ for all real numbers.

True or false:

(a) $(f \circ g)(x) = x$ for all real numbers.

(b) f and g are inverse functions.

6. The function f is described by the equation $f(x) = \sqrt{x+1}$ and the domain $[-1, \infty)$.

(a) What is the range of f ?

What are the coordinates of its x and y -intercepts?

(b) Is the function f one-to-one?

If so, find its inverse, giving the formula and the domain.

What are the coordinates of the x and y -intercepts of the inverse function of f ?

7. Determine whether $f(x) = \frac{4}{-5x+3}$ has an inverse function. If it does, find the inverse function.

If it has an inverse function:

What is the value of $f(1)$? Call it a .

Apply the function f inverse (written as f^{-1}) to a . Is the result OK?

8. Complete the square, getting the equation $y = \frac{1}{4}x^2 - 2x - 12$ into standard form, and sketch its graph. Plot the vertex, axis of symmetry, and all intercepts, labelling with both coordinates or the equation.
9. Find the largest area that a farmer can enclose by constructing a rectangular pen from 26 feet of fencing, if he uses a corner of his barn for two walls of the pen.



10. On the graph of the function $y = \log_2 x$, when the x -coordinate of point B is 32 times the x -coordinate of point A, what is the change in y ?
11. Given that $\log_b 2 = 3/4$, find $\log_b \frac{1}{2}$, $\log_2 b$, and (to two decimal places) b .
12. True or false:
- $\log(3.4 \times 13.4) = \log 3.4 + \log 13.4$.
 - $\log 2.5 \times \log 4 = \log 6.5$.
 - $\frac{\log \frac{1}{2}a - \log a}{\log \left(\frac{1}{2}\right)^{1/2}} = 2$ for all a such that $a > 0$.
 - $\log \left(\frac{1}{100}a^2\right) = 2 \log a - 2$ for a any non-zero real number.
 - The log of the quotient equals the difference of the logs.
13. True or false:
- $\frac{\log_a 5}{\log_a 2} = \log_2 5$.
 - $\frac{\log_a 5}{\log_a 2} = \frac{\log 5}{\log 2}$.
14. Decide if each statement is true or false. Then justify your answer by writing an equation.
- Multiplying two numbers then taking the log gives the same result as taking each log and then adding them.
 - Taking the logs of two numbers then dividing those two logs gives the same result as subtracting the two numbers then taking the log of that difference.
 - The product of $\log_a b$ and $\log_b a$ is always equal to 1.
15. In each of parts (a) through (h), approximate the logarithm, using the properties of logarithms, given $\log_b 2 \approx 1.098$, $\log_b 3 \approx 1.740$, and $\log_b 5 \approx 2.5495$.
- $\log_b 6$
 - $\log_b \frac{3}{5}$
 - $\log_b 125$
 - $\log_b \sqrt{3}$
 - $\log_b 20$
 - $\log_b (4b)^{-2}$
 - $\log_b (5b^2)$
 - $\log_b \sqrt[3]{2b}$
16. Solve the equation $\log x - \log(x - 1) = 2$.
Before solving, decide which x -values are valid substitutions into both of these logs.
17. Solve algebraically:
- $\log x + \log(x - 15) = 2$.
 - $\log x - \log(x - 15) = 2$.
 - $\log 24x - \log(1 + \sqrt{x}) = 2$.
18. First decide in which intervals all valid solutions must lie.
Then solve for x .
- $$\log_2 x - \log_2(2x - 1) = -3.$$
- Check your solutions in the original equation.

19. Solve algebraically: $\log_2 x + \log_2(1 - 3x) = -4$.

Hint: to solve the equation found after applying some log rules, just use the quadratic formula.

20. First decide in which intervals all valid solutions must lie.

Then solve for x .

$$\log_2 x + \log_2(1 - 2x) = -3.$$

Check your solutions in the original equation.

21. A population of fruit flies is increasing according to the law of exponential growth. At time 2 hours there are 2 pounds of flies and at time 32 hours there are 32 pounds of flies.

- Find the exact value of the doubling time. (No calculator is necessary.)
- True or false: at time 8 hours there were exactly 8 pounds of fruit flies.
- If false, about how many pounds of fruit flies were there at time 8 hours (to the nearest three-decimal accuracy or as an exact radical expression).
- At exactly what time will there be 64 pounds of fruit flies?

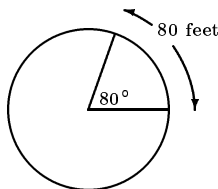
22. (a) Rewrite in radian measure as a fractional multiple of π and in degree measure: $3/16$ of a revolution.

(b) Rewrite in degree measure: $\frac{7\pi}{8}$.

(c) Rewrite in radian measure as a fractional multiple of π in lowest terms: 132° .

(d) Find the length of the arc on a circle of radius $150/\pi$ feet intercepted by a central angle of 150° .

23. A group has a banner that is 80 feet long. They wish to display it in the form of an arc of a circle that has angular measure of 80° . What is the radius of the circle needed for this layout? The answer may be left as $\frac{N}{\pi}$ feet.



24. A clock has a minute hand that is 4 feet long and an hour hand that is $2 \frac{1}{2}$ feet long.

- Find the angular velocity of the minute hand in radians per hour, in revolutions per hour, and in degrees per minute.
- Find the linear velocity of the tip of the minute hand in inches per minute.
- Find the angular velocity of the hour hand in radians per hour and in degrees per minute.
- At any moment, how fast is the angle between the two hands increasing or decreasing? Give answer in degrees per minute.

25. A right triangle has an acute angle θ with $\sec \theta = \frac{8}{7}$. Find the exact values of the other five trigonometric functions of θ , in fractional form. Some of the expressions will involve square roots; do not convert the square roots to decimals.

Then find the exact values of $\sec(90^\circ - \theta)$ and of $\csc^2 \theta - 1$, also in fractional form.

Hint. First sketch a right triangle corresponding to that secant. Next use the Pythagorean Theorem to determine the third side. Then find the other five trigonometric functions of θ .

For the other two values, use the appropriate trigonometric identities.

26. Find the hypotenuse of a right triangle in which an acute angle of 30° has an adjacent leg of 4 inches.

27. Simplify and reduce to an expression that contains at most one trig function.

(a) $\cos x(1 + \tan x)(1 - \tan x)$

(b) $\tan x \cos^2 x$

(c) $\cos^4 x - \sin^4 x$

(d) $\frac{1 + \cot^2 x}{\sin x}$

(e) $\frac{\sec x}{\csc x}$

(f) $\frac{\sec x}{\sin x}$

28. Find all solutions to $\cos \theta = \sqrt{3}/2$ in the interval $0 \leq \theta < 2\pi$.

29. Find all solutions to $\sin \theta = 0.669$ in the interval $0 \leq \theta < 180^\circ$.

Round off both answers to the nearest 0.01° .

30. Find all solutions to $\cos \theta = -0.26$ in the interval $0 \leq \theta < 180^\circ$.

Round off both answers to the nearest 0.01° .

31. (a) Find all solutions with $0 \leq x \leq 2\pi$ for $\sin 2x = -\cos x$.

(b) Graph $\sin 2x$ and $-\cos x$ on the same axes and indicate on your sketch the points corresponding to the solutions in part (a).

32. (a) Find all x between 0 and 2π for which $\sin 2x = -\sin x$.

(b) Sketch the graphs of $\sin 2x$ and $-\sin x$ on the same axes, indicating on your sketch the points corresponding to the solutions in part (a).

33. Find the period and the amplitude of

$$y = 5 \sin \left(2x - \frac{\pi}{4} \right).$$

Graph one period. Label with coordinates the endpoints of that period, the highest and lowest points, and all intercepts in that period.

State the phase fraction: the portion of a period that the graph was translated right (+) or left (-).

It might be less confusing with the 2 factored out of the expression in the parentheses.