Sample of Typical Final Examination Problems

Math 130 Precalculus for the December 22, 2017 Final Exam

No books, notes, or graphing calculators; scientific calculators permitted.
Show all work with at least four-decimal-place accuracy.

1. Find an equation of the line passing through the points $(8, -1)$ and $(-2, 3)$. Sketch the line and label both intercepts with their coordinates.

2. Write the standard form of the equation of a circle with endpoints of a diameter $(5, 6)$ and $(17, 15)$. State the coordinates of the center and the highest point, and find the length of the radius.

3. When $f(x) = 2x^2 + 3x - 1$ and $h \neq 0$, find $\dfrac{f(2+h) - f(2)}{h}$ and simplify the result.

4. In each case decide whether the function with the given rule is even, odd, or neither. Explain your reasoning or support your answer.

   (a) $f(x) = \dfrac{1}{x-1} + \dfrac{1}{x} + \dfrac{1}{x+1}$

   (b) $g(x) = |x+1|$

   (c) $h(x) = |x+3| - |x-3|$

   (d) $i(x) = \sqrt{(4-x)(4+x)}$

   (e) $j(x) = \sqrt{x+1}\sqrt{x-1}$, defined for real-valued outputs only.

5. Let $f(x) = \sqrt{x}$ for all $x \geq 0$ and $g(x) = x^2$ for all real numbers.

   True or false:

   (a) $(f \circ g)(x) = x$ for all real numbers.

   (b) f and g are inverse functions.

6. The function f is described by the equation $f(x) = \sqrt{x+1}$ and the domain $[-1, \infty)$.

   (a) What is the range of f?
   What are the coordinates of its x and y-intercepts?

   (b) Is the function f one-to-one?
   If so, find its inverse, giving the formula and the domain.
   What are the coordinates of the x and y-intercepts of the inverse function of f?

7. Determine whether $f(x) = \dfrac{4}{-5x+3}$ has an inverse function. If it does, find the inverse function.

   If it has an inverse function:

   What is the value of $f(1)$? Call it a.

   Apply the function f inverse (written as f^{-1}) to a. Is the result OK?

8. Complete the square, getting the equation $y = \frac{1}{4}x^2 - 2x - 12$ into standard form, and sketch its graph.
   Plot the vertex, axis of symmetry, and all intercepts, labelling with both coordinates or the equation.

9. On the graph of the function $y = \log_2 x$, when the x-coordinate of point B is 32 times the x-coordinate of point A, what is the change in y?

10. Given that $\log_b 2 = 3/4$, find $\log_b \frac{1}{2}$, $\log_2 b$, and (to two decimal places) b.

11. True or false:

    (a) $\log(3.4 \times 13.4) = \log 3.4 + \log 13.4$.

    (b) $\log 2.5 \times \log 4 = \log 6.5$.

    (c) $\dfrac{\log \frac{1}{2}a - \log a}{\log \left(\frac{1}{2}\right)^{1/2}} = 2$ for all a such that $a > 0$.

    (d) $\log \left(\frac{1}{100}a^2\right) = 2 \log a - 2$ for a any non-zero real number.

    (e) The log of the quotient equals the difference of the logs.

12. True or false:

    (a) $\dfrac{\log_a 5}{\log_a 2} = \log_2 5$.

    (b) $\dfrac{\log_a 5}{\log_a 2} = \dfrac{\log 5}{\log 2}$.

13. Decide if each statement is true or false. Then justify your answer by writing an equation.

    (a) Multiplying two numbers then taking the log gives the same result as taking each log and then adding them.

    (b) Taking the logs of two numbers then dividing those two logs gives the same result as subtracting the two numbers then taking the log of that difference.

    (c) The product of $\log_a b$ and $\log_b a$ is always equal to 1.

14. In each of parts (a) through (h), approximate the logarithm, using the properties of logarithms, given $\log_b 2 \approx 1.098$, $\log_b 3 \approx 1.740$, and $\log_b 5 \approx 2.5495$.

    (a) $\log_b 6$
    (b) $\log_b \frac{3}{5}$
    (c) $\log_b 125$
    (d) $\log_b \sqrt{3}$
    (e) $\log_b 20$
    (f) $\log_b (4b)^{-2}$
    (g) $\log_b (5b^2)$
    (h) $\log_b \sqrt[3]{2b}$

15. Solve the equation $\log x - \log(x - 1) = 2$.

    Before solving, decide which x-values are valid substitutions into both of these logs.

16. Solve algebraically:

    (a) $\log x + \log(x - 15) = 2$.

    (b) $\log x - \log(x - 15) = 2$.

    (c) $\log 24x - \log(1 + \sqrt{x}) = 2$.

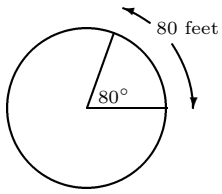
17. First decide in which intervals all valid solutions must lie.

    Then solve for x.

    $$\log_2 x - \log_2(2x - 1) = -3.$$

    Check your solutions in the original equation.

18. A group has a banner that is 80 feet long. They wish to display it in the form of an arc of a circle that has angular measure of 80°. What is the radius of the circle needed for this layout? The answer may be left as $\frac{N}{\pi}$ feet.



19. A clock has a minute hand that is 4 feet long and an hour hand that is 2 1/2 feet long.

    (a) Find the angular velocity of the minute hand in radians per hour, in revolutions per hour, and in degrees per minute.

    (b) Find the linear velocity of the tip of the minute hand in inches per minute.

    (c) Find the angular velocity of the hour hand in radians per hour and in degrees per minute.

    (d) At any moment, how fast is the angle between the two hands increasing or decreasing? Give answer in degrees per minute.

20. Find the hypotenuse of a right triangle in which an acute angle of 30° has an adjacent leg of 4 inches.

21. Find all solutions to $\cos\theta = \sqrt{3}/2$ in the interval $0 \le \theta < 2\pi$.

22. Find all solutions to $\sin\theta = 0.669$ in the interval $0 \le \theta < 180°$.

    Round off both answers to the nearest 0.01°.

23. Find all solutions to $\cos\theta = -0.26$ in the interval $0 \le \theta < 180°$.

    Round off both answers to the nearest 0.01°.

24. (a) Find all solutions with $0 \le x \le 2\pi$ for $\sin 2x = -\cos x$.

    (b) Graph $\sin 2x$ and $-\cos x$ on the same axes and indicate on your sketch the points corresponding to the solutions in part (a).

25. (a) Find all x between 0 and 2π for which $\sin 2x = -\sin x$.

    (b) Sketch the graphs of $\sin 2x$ and $-\sin x$ on the same axes, indicating on your sketch the points corresponding to the solutions in part (a).

26. Find the period and the amplitude of

$$y = 5\sin\left(2x - \frac{\pi}{4}\right).$$

    Graph one period. Label with coordinates the endpoints of that period, the highest and lowest points, and all intercepts in that period.

    State the phase fraction: the portion of a period that the graph was translated right (+) or left (−).

    It might be less confusing with the 2 factored out of the expression in the parentheses.

27. Use the Law of Cosines to solve the triangle with sides of lengths 2 and $5\sqrt{3}$, and an included angle of 30° between them. Find the third side and the other two angles. (Round angles to three decimal places.)

    Hint. After finding the third side ask yourself: Which is the longest side? So the largest angle will be between which two sides? Which is the shortest side? So the smallest angle will be between which two sides?