Sample of Typical Final Examination Problems

Math 130 Precalculus for the May 19, 2017 Final Exam

No books, notes, or graphing calculators; scientific calculators permitted.
Show all work with at least four-decimal-place accuracy.

1. Find an equation of the line passing through the points $(8, -1)$ and $(-2, 3)$. Sketch the line and label both intercepts with their coordinates.

2. Find the slope-intercept equation of the line with slope $-2/3$ and x-intercept 6.

3. Let A be the point with coordinates $(13.5, 4.5)$ and B be the point with coordinates $(22.5, 44.5)$.

   (a) For the line segment AB: find its length, the coordinates of its midpoint, and its slope.
   (b) Find an equation of the perpendicular bisector of AB.

4. Write the standard form of the equation of a circle with endpoints of a diameter $(5, 6)$ and $(17, 15)$. State the center and the radius.

5. When $f(x) = 2x^2 + 3x - 1$ and $h \neq 0$, find $\dfrac{f(2+h) - f(2)}{h}$ and simplify the result.

6. In each case decide whether the function with the given rule is even, odd, or neither. Explain your reasoning or support your answer.

   (a) $f(x) = \dfrac{1}{x-1} + \dfrac{1}{x} + \dfrac{1}{x+1}$
   (b) $g(x) = |x+1|$
   (c) $h(x) = |x+3| - |x-3|$
   (d) $i(x) = \sqrt{(4-x)(4+x)}$
   (e) $j(x) = \sqrt{x+1}\sqrt{x-1}$, defined for real-valued outputs only.

7. Graph $y = x^3$. Then write an equation of that graph shifted three units to the left and three units down.

8. Graph $y = \dfrac{1}{x}$. Then write an equation of that graph shifted four units to the left and four units down.

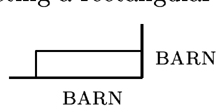
9. The function f is described by the equation $f(x) = \sqrt{x+1}$ and the domain $[-1, \infty)$.

   (a) What is the range of f?
   What are the coordinates of its x and y-intercepts?
   (b) Is the function f one-to-one?
   If so, find its inverse, giving the formula and the domain.
   What are the coordinates of the x and y-intercepts of the inverse function of f?

10. Complete the square, getting the equation $y = \frac{1}{4}x^2 - 2x - 12$ into standard form, and sketch its graph. Plot the vertex, axis of symmetry, and all intercepts, labelling with both coordinates or the equation.

11. (a) Graph the equation $y = \frac{1}{4}x^2 - 2x - 12$ when $x \geq 4$.
    i. Does this equation describe y as a function of x?
    (b) Is the inverse of the relation described in part (a) a function?
    i. If so, find a closed-form formula for the inverse function and graph it.

12. Find the largest area that a farmer can enclose by constructing a rectangular pen from 26 feet of fencing, if he uses a corner of his barn for two walls of the pen.



13. Adding twice one number to twice another number gives a total of 34.
    What is the largest the product of those two numbers could possibly be?

14. On the graph of the function $y = \log_2 x$, when the x-coordinate of point B is 32 times the x-coordinate of point A, what is the change in y?

15. Find $\log_9 3\sqrt{3}$ as an exact fraction or an exact decimal.

16. Given that $\log_b 2 = 3/4$, find $\log_b \frac{1}{2}$, $\log_2 b$, and (to two decimal places) b.

17. Find the equation of the straight line through the points on the graph of $y = \log_4 x$ where $x = 8$ and where $x = 32$.

18. Solve the equation $\log x - \log(x-1) = 2$.

    Before solving, decide which x-values are valid substitutions into both of these logs.

19. Solve algebraically:

    (a) $\log x + \log(x-15) = 2$.

    (b) $\log x - \log(x-15) = 2$.

    (c) $\log 24x - \log(1+\sqrt{x}) = 2$.

20. First decide in which intervals all valid solutions must lie.

    Then solve for x.

$$\log_2 x - \log_2(2x-1) = -3.$$

    Check your solutions in the original equation.

21. True or false:

    (a) $\log(3.4 \times 13.4) = \log 3.4 + \log 13.4$.

    (b) $\log 2.5 \times \log 4 = \log 6.5$.

    (c) The log of the quotient equals the difference of the logs.

22. Decide if each statement is true or false. Then justify your answer by writing an equation.

    (a) Multiplying two numbers then taking the log gives the same result as taking each log and then adding them.

    (b) Taking the logs of two numbers then dividing those two logs gives the same result as subtracting the two numbers then taking the log of that difference.

    (c) The product of $\log_a b$ and $\log_b a$ is always equal to 1.

23. True or false. (In each part assume that all three numbers are positive.)

    (a) $a^{\log_b c}$ and $b^{\log_a c}$ are equal.

    (b) $a^{\log_b c}$ and $c^{\log_b a}$ are equal.

24. Either prove that $a^{\log b}$ and $b^{\log a}$ are equal or give an example of two numbers for which the expressions are not equal.

    (Here's an example: pick $a = 3$ and $b = 100$, note that $a^{\log b} = 3^2 = 9$, and then evaluate $b^{\log a}$.)

25. The number of bacteria present in a culture is increasing according to the law of exponential growth. After 3 hours there are 100 bacteria and after 7 hours there are 400 bacteria. How many bacteria will there be after 8 hours?

26. The number of bacteria present in a culture is increasing according to the law of exponential growth. After 1 hour there are 200 bacteria and after 2 hours there are 320 bacteria. How many bacteria will there be after 3 hours?

    Is the answer 440 bacteria, assuming an increase of 120 bacteria for each hour of time?

27. Find the length of the arc on a circle of radius 3 meters intercepted by a central angle of $150°$.

28. Find the length of the arc on a circle of radius $150/\pi$ feet intercepted by a central angle of $150°$.

29. A clock has a minute hand that is 4 feet long and an hour hand that is 2 1/2 feet long.

    (a) Find the angular velocity of the minute hand in radians per hour, in revolutions per hour, and in degrees per minute.

    (b) Find the linear velocity of the tip of the minute hand in inches per minute.

    (c) Find the angular velocity of the hour hand in radians per hour and in degrees per minute.

    (d) At any moment, how fast is the angle between the two hands increasing or decreasing? Give answer in degrees per minute.

30. Find $\cos 30°$, $\arccos\left(\frac{\sqrt{3}}{2}\right)$, and $\arccos\left(-\frac{\sqrt{3}}{2}\right)$. Then find $\sin 30°$ and $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$.

    In each part, find a formula that connects the expressions and show that it holds for the given values.

    (a) $\cos 30°$ and $\sin 30°$.

    (b) $\arccos\left(-\frac{\sqrt{3}}{2}\right)$ and $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$.

    (c) $\cos\left[\arccos\left(-\frac{\sqrt{3}}{2}\right)\right]$ and $-\frac{\sqrt{3}}{2}$.

    (d) $\arccos\left(-\frac{\sqrt{3}}{2}\right)$ and $\arccos\left(\frac{\sqrt{3}}{2}\right)$.

31. Find $\tan\theta$ when $\sec\theta = \sqrt{10}$ and $\sin\theta$ is negative.

32. Find $\cot\theta$ when $\csc\theta = \frac{1}{2}\sqrt{29}$ and $\cos\theta$ is positive.

33. Let $\sin\theta = r$, where $-1 \le r \le 1$.
    Find, in terms of r:

    (a) $\sec 2\theta$. Assume that $r \neq \pm\sqrt{2}/2$.

    (b) $\sin(\theta + \frac{\pi}{4})$, assuming that θ is between $-\pi/2$ and $\pi/2$.

    (c) An expression that gives a value for θ in the interval $\pi/2 \le \theta < 3\pi/2$.
    Hint: This will involve an inverse trig function of r.

34. Assume that the cosine of $28.1255° = \sqrt{7}/3$.
    Without using a calculator find the values of:

    (a) $\cos 61.8745°$. (b) $\cos 151.8745°$. (c) $\cos 331.8745°$. (d) $\cos 208.1255°$. (e) $\sin 151.8745°$.

35. Find the hypotenuse of a right triangle in which an acute angle of $30°$ has an adjacent leg of 4 inches.

36. Find all solutions to $\cos\theta = \sqrt{3}/2$ in the interval $0 \le \theta < 2\pi$.

37. Find all solutions to $\sin\theta = 0.669$ in the interval $0 \le \theta < 180°$.
    Round off both answers to the nearest $0.01°$.

38. Find all solutions to $\cos\theta = -0.26$ in the interval $0 \le \theta < 180°$.
    Round off both answers to the nearest $0.01°$.

39. Simplify and reduce to an expression that contains at most one trig function.

    (a) $\cos x(1 + \tan x)(1 - \tan x)$

    (b) $\tan x \cos^2 x$

    (c) $\cos^4 x - \sin^4 x$

    (d) $\dfrac{1 + \cot^2 x}{\sin x}$

    (e) $\dfrac{\sec x}{\csc x}$

    (f) $\dfrac{\sec x}{\sin x}$

40. Find the numerical value of $\cos 2\theta - 2\cos^2\theta$.

41. Find all solutions of $3\cos 2x = 2\cos^2 x$ in the interval $[0, 2\pi)$. (Get the algebraically-assisted, exact result.)

42. (a) Find all solutions with $0 \le x \le 2\pi$ for $\sin 2x = -\cos x$.
    (b) Graph $\sin 2x$ and $-\cos x$ on the same axes and indicate on your sketch the points corresponding to the solutions in part (a).

43. (a) Find all x between 0 and 2π for which $\sin 2x = -\sin x$.
    (b) Sketch the graphs of $\sin 2x$ and $-\sin x$ on the same axes, indicating on your sketch the points corresponding to the solutions in part (a).

44. (a) Use an appropriate half-angle formula to find the exact value of $\sin 15°$.
    (b) Use a formula for the sine of the difference of the angles 45° and 30° to obtain an exact expression for the sine of 15°.
    (c) Those expressions are not the same! What gives?

45. Find the exact value of $\cos 67\frac{1}{2}°$.

46. Find the exact value of $\cos 15°$.

47. Given that $\cos u = -2/5$ with $\pi/2 < u < \pi$, find the exact value of $\cos 4u$ using double-angle formulas, as needed. *Hint:* use fractions, not decimals.

48. Given that $\cos u = -2/3$ with $\pi/2 < u < \pi$, find the exact value of $\cos 4u$ using double-angle formulas, as needed. *Hint:* use fractions, not decimals.

49. Given that $\cos u = \sqrt{6}/6$ with $3\pi/2 < u < 2\pi$, find the exact value of $\cos 4u$ using double-angle formulas, as needed. Don't use a calculator. *Hint:* use fractions, not decimals.

50. Use the Law of Cosines to solve the triangle with sides of lengths 1 and $\sqrt{3}$, and an included angle of 30° between them.

    Find the third side and the other two angles.

51. Use the Law of Cosines to solve the triangle with sides of lengths 2 and $5\sqrt{3}$, and an included angle of 30° between them. Find the third side and the other two angles. (Round angles to three decimal places.)

    Hint. After finding the third side ask yourself: Which is the longest side? So the largest angle will be between which two sides? Which is the shortest side? So the smallest angle will be between which two sides?

52. Find $\sin\theta$ when $\cot\theta = -\frac{24}{7}$ and $\cos\theta < 0$.

    In which quadrant of the unit circle is the terminal point of θ?

53. Find $\cos\theta$ when $\tan\theta = \frac{\sqrt{13}}{6}$ and $\sin\theta < 0$.

    In which quadrant of the unit circle is the terminal point of θ?

54. Assume that the cosine of 75.5225° is 1/4. (This is actually an approximation.)

    Find without a calculator, using the sum, double-angle, and half-angle formulas:

    (a) the cosine of 226.5675° as a rational number.
    Suggestion: first find the sine of 75.5225°, and then find sine and cosine of 151.0450°.
    (b) the cosine of 120.5225°, leaving it in radical form.
    (c) the sine and the cosine of $\frac{1}{2}(75.5225°)$, simplified to rational multiples of $\sqrt{6}$ and $\sqrt{10}$.