Sample of Typical Final Examination Problems

Math 130 Precalculus for the December 19, 2014 Final Exam

No books, notes, or graphing calculators; scientific calculators permitted.
Show all work with at least four-decimal-place accuracy.

1. Write the standard form of the equation of a circle with endpoints of a diameter $(5, 6)$ and $(17, 15)$. State the center and the radius. Find the coordinates of the highest point on the circle. Is the point $(5, 15)$ inside, outside, or on the circle? What about the point $(5, 17)$?

2. Find an equation of the line passing through the points $(8, -1)$ and $(-2, 3)$. Sketch the line and label both intercepts with their coordinates.

3. When $f(x) = 2x^2 + 3x - 1$ and $h \neq 0$, find $\dfrac{f(x+h) - f(x)}{h}$ and simplify the result.

4. For $f(x) = \sqrt{x}$ and $g(x) = x^4 - 6x^2 + 9$, find and simplify $f \circ g$ and $g \circ f$. The simplified results will have no square roots.

   Find the domain of each function and the domain of each composite function (over the real numbers).

5. Determine whether $f(x) = \dfrac{4}{-5x+3}$ has an inverse function. If it does, find the inverse function.

6. Write the quadratic function $f(x) = \frac{1}{4}x^2 - 2x - 12$ in standard form and sketch its graph.

   Plot the vertex, axis of symmetry, and all intercepts, labelling with both coordinates or the equation.

7. Consider ordered pairs of real numbers for which half the first number plus the second number equals 5. Of all such ordered pairs, which one has the largest product?

   For that maximizing pair, find the first and second numbers and their product.

8. Let $w = \log_2 a$.

   Find an expression in terms of w for:

   (a) $\log_2 a^4$
   (b) $\log_2 (4a^2)$
   (c) $\log_2 (4a)^2$
   (d) $\log_2 (\sqrt[4]{a})$
   (e) $[\log_2 4a]^2$
   (f) $\sqrt{\log_2 a^2}$

9. In each of parts (a) through (h), approximate the logarithm, using the properties of logarithms, given $\log_b 2 \approx 1.098$, $\log_b 3 \approx 1.740$, and $\log_b 5 \approx 2.5495$.

   (a) $\log_b 6$
   (b) $\log_b \frac{3}{5}$
   (c) $\log_b 125$
   (d) $\log_b \sqrt{3}$
   (e) $\log_b 20$
   (f) $\log_b (4b)^{-2}$
   (g) $\log_b (5b^2)$
   (h) $\log_b \sqrt[3]{2b}$

10. (a) Simplify to an exact rational number: $\log_2 \left(\sqrt[3]{128}\right)$.

    (b) How is $\log_2 64a^2$ related to $\log_2 a$?

11. (a) Simplify to a rational number:
$$\log_2\left(16\sqrt[3]{1/4}\right).$$
(b) Simplify to a rational number or to an exact decimal:
$$\left[1+3\log_2\left(\sqrt[4]{2}\right)\right]^2.$$

12. Solve algebraically:
    (a) $\log x + \log(x-15) = 2.$
    (b) $\log x - \log(x-15) = 2.$
    (c) $\log_8(x+1) - \log_8(x-3) = \frac{1}{3}.$
    (d) $\log 24x - \log(1+\sqrt{x}) = 2.$

13. First decide in which intervals all valid solutions must lie.

    Then solve for x.
$$\log_2 x + \log_2(1-2x) = -3.$$
    Check your solutions in the original equation.

14. Decide if each statement is true or false. Then justify your answer by writing an equation.
    (a) Multiplying two numbers then taking the log gives the same result as taking each log and then adding them.
    (b) Taking the logs of two numbers then dividing those two logs gives the same result as subtracting the two numbers then taking the log of that difference.

15. The number of bacteria present in a culture is increasing according to the law of exponential growth. After 3 hours there are 100 bacteria and after 7 hours there are 400 bacteria. How many bacteria will there be after 8 hours?

16. The number of bacteria in a culture is increasing according to the law of exponential growth. The initial population is 230 bacteria, and the population after 7 hours is double the population after 1 hour. How many bacteria will there be after 4 hours?

17. (a) Rewrite in radian measure as a multiple of π: (i) $130°$ (ii) $-60°$
    (b) Rewrite in degree measure: (i) $\dfrac{3\pi}{2}$ (ii) $\dfrac{5\pi}{4}$ (Do not use a calculator.)
    (c) Find the length of the arc on a circle of radius 3 meters intercepted by a central angle of $150°$.

18. A carousel with a 50-foot diameter makes 4 revolutions per minute.
    (a) Find the angular speed of the carousel in radians per minute.
    (b) Find the linear speed (in feet per minute) of the platform rim of the carousel.
    (c) If an individual sat at the rim and rode for an hour, how many miles (rounded to two decimal places) would he have travelled?

19. A right triangle has an acute angle θ with $\sec\theta = \frac{8}{7}$. Find the exact values of the other five trigonometric functions of θ, in fractional form.

    Then find the exact values of the sine and cosine of 2θ and the cosine of 3θ, also in fractional form.

    Hint. First sketch a right triangle corresponding to that secant. Next use the Pythagorean Theorem to determine the third side. Then find the other five trigonometric functions of θ.

    For the double and triple angles, use the appropriate formulas.

20. A right triangle has an acute angle θ with $\sin\theta = \frac{1}{6}$. Find the exact fractional value of $\cos 4\theta$. *Do not* use a calulator at all to find this answer. Show how you did this. Show all of your work, including each step.

    Now check your answer, using a calculator.

    - Find θ in degrees.
    - Find 4θ in degrees.
    - Find $\cos 4\theta$ in decimal form, using the calculator.
    - Put your earlier fractional answer in decimal form. Does this agree with the calculator's value of $\cos 4\theta$?

21. Given that $\cos u = -4/5$ with $\pi/2 < u < \pi$, find the exact values of $\sin 3u$ and $\cos 3u$ using the double-angle formulas and the sum formulas, as needed. Don't use a calculator. *Hint:* use fractions, not decimals.

22. Given that $\cos u = 3/\sqrt{10}$, find the exact value of $\cos 8u$. The size of u is unspecified; it could be more than $\pi/2$. Don't use a calculator. Show each step of your work. You might need the fact that $25^2 = 625$.

    Hint: use fractions, not decimals.

    As a bit of a check:

    Does the cosine of 341.565° approximately equal $3/\sqrt{10}$?

    Is the cosine of eight times that angle—when rounded off to four decimal places—the same as the decimal equivalent of your answer?

23. Derive formulas for $\sin 3x$ in terms of $\sin x$, and $\cos 3x$ in terms of $\cos x$.

    It will be necessary to take the initial answer obtained using the sum and double-angle formulas and substitute a trig identity to get an equivalent form in the one trig function.

    Select an angle of your choosing and verify with a calculator that the formulas work for it.

24. Simplify and reduce to an expression that contains at most one trig function.

    (a) $\cos x(1 + \tan x)(1 - \tan x)$

    (b) $\tan x \cos^2 x$

    (c) $\cos^4 x - \sin^4 x$

25. Find all solutions of $3\cos 2x = 2\cos^2 x$ in the interval $[0, 2\pi)$. (Get the algebraically-assisted, exact result.)

26. Use the Law of Cosines to solve the triangle with sides of lengths 1 and $\sqrt{3}$, and an included angle of 30° between them.

27. Use the Law of Cosines to solve the triangle with sides of lengths 2 and $5\sqrt{3}$, and an included angle of 30° between them. (Round angles to three decimal places.)

    Hint. After finding the third side ask yourself: Which is the longest side? So the largest angle will be between which two sides? Which is the shortest side? So the smallest angle will be between which two sides?

    Find the exact area of this triangle. Leave it as multiple of $\sqrt{3}$.

    Hint. Drop a perpendicular from the opposite vertex to the side of length $5\sqrt{3}$.