Trig Review for Dec. 2016 final

Math 130 *Kovitz* Fall 2016

The final is on Friday, December 16, at 6:30 pm.

Mr. Kovitz' review: Thurs., Dec. 15, from 12 noon to 2:45 p.m. in W-1-0041.

1. Find the period and the amplitude and the y-intercept of

$$y = 4\sin\left(2x + \frac{2\pi}{3}\right).$$

Graph one period. Label with coordinates the endpoints of that period, the highest and lowest points, and all intercepts in that period.

State the phase fraction: the portion of a period that the graph was translated right (+) or left (−).

This problem might be less confusing with the 2 factored out of the expression in the parentheses.

Here are some ideas to make the problem more manageable.

- Find the amplitude, period, and phase fraction. For the phase fraction, take minus the original horizontal-shift term divided by the initial period of the function $y = \sin x$ (which will be 2π).
- Factor out the mutliplier of the x term.
- Ignore the shift and graph the first period of the parent function, which is in this case of the form $A\sin(ax)$. Plot with labels on your graph all x-intercepts and all highest and lowest points in that period.
- Recheck the phase fraction by dividing minus the new shift by the new period. It should come out to be the same.
- Move the new graph by the indicated amount.
- (optional) Add the y-intercept to the graph and expand the domain of the new graph to include it. Always calculate the y-intercept by substituting $x = 0$ in the original unfactored form of the equation, if if is available.

2. Apply the Law of Cosines to a triangle with sides of lengths $2\sqrt{3}$ and 5, and an included angle of 30° between them.

Find the length of the third side. Leave this answer as a radical; do not find the decimal value. No calculator is needed.

To three decimals, find the size of the angle opposite the longest side. A calculator will be needed.

Apply the Law of Cosines to find the size of the angle opposite the second longest side.

Several questions to affirm that the result is reasonable:

- Does the shortest side have the smallest angle opposite it?
- Does the longest side have the largest angle opposite it?
- Do the three angles add up to 180.000°?
- Should the two shorter sides of every triangle add up to more than the longest side? Is that the case here?

3. Solve the equation $\sin \frac{1}{2}x = \cos x$ for $0 \le x < 2\pi$.

   (a) graphically,

   and

   (b) symbolically (algebraically).

   Explain why your solutions are not the same, and fix up the result.

4. In a circle the arc opposite a central angle of 12° is 3.2 yards long.

   Find the length of the radius. The answer may be left in the form $\frac{n}{\pi}$ feet.

5. Let $\sin\theta = w$, where $-1 \le w \le 1$ and $w \ne 0$.

   Find, in terms of w:

   (a) $\cos^2\theta$.
   (b) $\csc^2\theta$.
   (c) $\cot^2\theta$.
   (d) $1 + \tan^2\theta$.
   (e) $\cos 2\theta$.
   (f) $\csc(\theta + \pi)$.
   (g) $\cos\left(\frac{\pi}{2} - \theta\right)$.

6. Let $\cos\theta = w$, with $-\pi \le \theta \le 0$.

   Find, in terms of w, $\sin \frac{1}{2}\theta$.

7. Let $\cos\theta = w$, where $-1 < w < 1$.

   Find both solutions for θ between 0 and 2π, each solution expressed in terms of the arccosine function. Hint: draw the unit circle.

8. Assume that the cosine of 16.2602° is 24/25 (approximately).

   Find without a calculator, using the sum, double-angle, and half-angle formulas:

   (a) an approximation of the sine of 32.5204° as a rational number. Suggestion: first find the sine of 16.2602°.
   (b) an approximation of the cosine of 32.5204° as a rational number.
   (c) an approximation of the cosine of 76.2602°, leaving $\sqrt{3}$ as a radical in the expression. Do not find the answer as a decimal.
   (d) an approximation of the sine and the cosine of 8.1301°, simplified to rational multiples of $\sqrt{2}$.