# Review and Further Examples for Calculus I

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1. Find the angle of inclination of the line $y = -2x + 11$.

   The angle of inclination is the smallest positive angle from the $x$-axis to the line.

2. Write an equation for the straight line that has $y$-intercept $-6$ and $x$-intercept 2.

3. A particle starts at $A(-4, 7)$ with $\Delta x = 8$ and $\Delta y = -12$.

   Find the new position.

4. Find
   $$\frac{f(x+h) - f(x)}{h}, \quad \text{where } h \neq 0,$$
   for the function $f(x) = 2x - 3x^2$.

5. Find the domain and range of the following functions:

   (a) $f(x) = x^2 + 6$.

   (b) $f(x) = \sqrt{1 - 2(x+6)^2}$.

   (c) $f(x) = \sqrt{36 - x}$. Also sketch the graph of $f$.

6. Find a function whose graph is the line segment joining the points $(-6, -8)$ and $(4, -3)$.

7. Express the surface area of a sphere as a function of its volume.

   Check your result for the sphere with volume equal to $\frac{500}{3}\pi$ cubic inches. What would the radius be in that case?

8. Let $f(x) = x - 3$ and let $g(x) = 2x^2 - 11$.

   (a) Find the functions $f \circ f$ and $f \circ g$ and their domains.

   (b) Find $(f \circ f)(1)$, $(f \circ f)(-1)$, $(f \circ g)(1)$, and $(f \circ g)(-1)$.

   (c) Determine whether each of $f \circ f$ and $f \circ g$ are even, odd, or neither.

   (d) Find a formula for the inverse of $f$. Is that inverse a function?

9. Let $f(x) = x - 7$, $g(x) = 3x$, and $h(x) = x^2$.

   Express $j(x) = x - 21$ as a composite involving one or more of $f$, $g$, and $h$.

10. Identify the type of curve and sketch the graph, labeling all intercepts with their coordinates. If the graph is a circle, label the center with its coordinates and find the radius.

    (a)
    $$(x+7)^2+(y-2)^2=8$$

    (b)
    $$x^2+y^2+18x-10y+90=0$$

11. Graph each function, not by plotting points, but by starting with the graph of one of the standard functions, and then applying the appropriate transformations.

    (a)
    $$y=(x+11)^3+8$$
    Where, if anywhere, is the geometrically distinguished point of the graph?

    (b)
    $$y=-\frac{1}{3}x^2+x-\frac{7}{4}$$
    Label with coordinates any geometrically distinguished point of the graph.

    (c)
    $$y=\sin\left(\frac{x}{4}-\frac{\pi}{6}\right)$$
    Find the period and, for the interval $0\le x\le 10\pi$, give the coordinates of the $x$- and $y$-intercepts and the highest and lowest points.

    Where in the interval $0\le x\le 10\pi$ is $y=\frac{\sqrt{3}}{2}$?

    (d)
    $$y=2-4\sin\pi x$$
    Find the period and, on the interval $0\le x\le 3$, give the coordinates of the $x$- and $y$-intercepts and the highest and lowest points.

12. Find an inequality whose graph is the finite area between the curves $y=x$ and $y=x^2-6$ and state the domain and range of that inequality.

13. Solve $|3x-8|+|x+4|=11$.

14. Graph $f(x)=|x+1|+|x+2|+|x+3|$.

15. Rewrite $-9<x<4$ as an absolute-value inequality.

16. Solve for $x$.

    (a) $\sqrt{2x+3}=3$
    (b) $|x-3|=8$
    (c) $2\ge|x-7|$
    (d) $x^3-3x^2\ge 10x$
    (e) $\dfrac{x-3}{x^2-4}\ge 0$

17. Solve the inequality $|x+5|+|7-5x|<11$.

18. (a) Find an exact expression for $\cos 75^\circ$ by using a an addition formula, starting with any two first-quadrant angles that have well-known sines and cosines.

    (Leave the expression in an exact form in terms of square roots.)

    (b) Find another exact expression for $\cos 75^\circ$ by means of the half-angle identity.

    (Leave the expression in an exact form, using square roots when needed.)

    (c) Are the expressions in parts (a) and (b) different?

    Using a calculator, obtain the numerical value of each expression and of $\cos 75^\circ$ directly.

    Do all three have the same value?

    (d) If the expressions in parts (a) and (b) are equal, verify that fact algebraically.

19. (a) The radius of the earth is about 3960 miles. What length of ribbon would you need to wrap around the earth at the equator?

    (b) How much more ribbon would you need if you raised the ribbon one foot above the earth.

20. A metal railroad track one mile long is anchored on both ends and hinged at the midpoint. On a hot day the length expands by one foot, bowing upward and forming two right triangles each with one leg of length $\frac{1}{2}$ mile.

    About how high would the center of the rail rise above the ground?

21. A Tibetan monk leaves the monestary at 7:00 A.M. and takes his usual path to the top of the mountain, arriving at 7:00 P.M. The following morning, he starts at 7:00 A.M. at the top and takes the same path back, arriving at the monestary at 7:00 P.M.

    Show that there is a point on the path that the monk will cross at exactly the same time of day on both days.

22. You have two cups in front of you: one holds coffee and one holds milk. You take a teaspoon of milk from the milk cup and stir it into the coffee cup. Next, you take a teaspoon of the mixture in the coffee cup and put it back into the milk cup. After two transfers, is there more milk in the coffee cup or more coffee in the milk cup?

23. Khai works 8 miles from his home and drives home each day at the same constant speed. At exactly the same time as he leaves for home, Thuy leaves the house to jog at four miles per hour toward his office. If Khai were to drive ten miles per hour faster each day, he would pass Thuy four minutes earlier.

    How fast does Khai currently drive home? After how many minutes does he now pass Thuy on his way home?

# Useful Formulas and Definitions

## Composition of Functions:

Given two functions $f$ and $g$, the **composite function** $f \circ g$ (also called the **composition** of $f$ and $g$) is defined by

$$(f \circ g)(x) = f(g(x))$$

## Vertical and Horizontal Shifts:

Given $c > 0$, in each of the following cases to obtain the graph of the given equation, shift the graph of $y = f(x)$ $c$ units

$y = f(x) + c$: upward.
$y = f(x) - c$: downward.
$y = f(x - c)$: to the right.
$y = f(x + c)$: to the left.

## Vertical and Horizontal Stretching and Shrinking:

Given $c > 1$, in each of the following cases to obtain the graph of the given equation from the graph of $y = f(x)$, use a factor of $c$, and

$y = cf(x)$: stretch vertically.
$y = (1/c)f(x)$: compress vertically.
$y = f(cx)$: compress horizontally.
$y = f(x/c)$: stretch horizontally.

## Reflecting;

In each of the following cases to obtain the graph of the given equation, reflect the graph of $y = f(x)$ about the

$y = -f(x)$: $x$-axis.
$y = f(-x)$: $y$-axis.