Solutions to Practice for Test 3, Math 125

(The test will be on May 12)

Math 125 *Kovitz* Spring 2025

On the test, show your work.

1. (B) 71.53% to 76.07

   (a) Set up a 0–1 box where 1 is on the tickets for Democrats and 0 on the others.
   (b) Find the sample percent: $(1107/1500) \times 100\% = 73.8\%$.
   (c) Use the bootstrap to estimate the fraction of 1's in the box: 1107/1500 or 0.738. The fraction of 0's in the box will be 0.262.
   (d) Estimate the SD of the box by the shortcut: $\sqrt{0.738 \times 0.262} = \sqrt{0.193356} = 0.43972$.
   (e) Estimate the SE for the percentage: $\frac{\text{SD of box}}{\sqrt{\text{no. of draws}}} \times 100\% = \frac{0.43972}{\sqrt{1500}} \times 100\% = \frac{43.972\%}{38.73} = 1.135\%$.
   (f) Write 95%-confidence interval as sample percent plus or minus two SEs.
   (g) Answer is: $73.8\% \pm 2.27\%$. That's 73.8%-2.27% to 73.8%+2.27%, or 71.53% to 76.07%, answer (B).

2. (E) 98.76%

   The average of the draws is the same as the sample average.

   The expected value for the average of the draws is equal to the average of the box, here 50. (See the box on page 410.)

   The standard error for the average is (using the technical note on page 415) the SD of the box, divided by the square root of the number of draws.

   Here it gives: $8/\sqrt{25} = 8/5 = 1.6$.

   A sample of size 25 should be marginally large enough for the normal approximation to apply.

   The standard units for a given sample average is found with the formula:

   $$z = \frac{\text{given sample average} - \text{EV for sample average}}{\text{SE for sample average}}.$$

   Find the standard units for 46 and 54: $(46 - 50)/1.6 = -2.5$ and $(54 - 50)/1.6 = 2.5$.

   The area under the normal curve from -2.5 to $+2.5$ is just the table entry: 98.76%, answer (E).

3. We do not know the SD of the box. However the largest an SD of a 0-1 box can be is 0.5. So assume the worst case and use 0.5 for the SD.

   Use the technical note on page 362 to find a useful formula for the SE for the percentage.

   SE % $= \left[(\text{SD of the box})/\sqrt{\text{no. of draws}} \right] \times 100\%$.

   Now $2\% = 50\%/\sqrt{n}$.

   Solve algebraically:

   $\sqrt{n} = 50\%/\ 2\%$.

   $\sqrt{n} = 25$.

   Square both sides to get: $n = 625$.

   Answer (B) 625

   Note: The problem could be answered without the algebraic solution by taking the largest possible SD (0.5), and plugging in each of the five possible answers into the formula for the standard error for the percentage, and seeing which of the five gives 2%.

4. Answer: (D)

   To show that statement (A) is true, see the first paragraph in Section 2 of Chapter 23, found on page 415.

   For statements (B) and (C), start with the box on page 416.

   The theory of the Gauss model is presented in the box on page 450.

   Read the first paragraph on page 451 and use the box on page 451.

   Statement (C) is shown to be true by reference to the first sentence in the box on page 67.

   For statement (B): to get the standard error for the sample average, use technical note (ii), found at the top of page 415. The estimate, 300,007, is the sample average, so it will be off the exact speed of light (the population average) by 2, the standard error for the sample average, making statement (B) true.

   Starement (D) is false because each measurement is off by the SD of the error box (10) and not by the standard error for the average of the draws (2).

   Statement (E) is true by the formula in the 3rd bullet on page 381, applied to averages.

5. Rolling the die is like drawing at random with replacement 4800 times from a 0–1 box, with 0 = non-ones and 1 = ones. The fraction of 1's in the box is unknown.

   Null hypothesis: this fraction equals 1/6.

   Alternative: the fraction is smaller than 1/6. (We observe that 761 is somewhat smaller than 800, the expected value.)

   The number of one spots is like the sum of the draws.

   $z = -1.50$, $P \approx 7\%$, fair. Answer: (C)

   Calculation: The average of the box = 1/6 and the SD of box = $\sqrt{1/6 \times 5/6} = \sqrt{5}/6 \approx 0.372678$.

   So the EV sum = $4800 \times 1/6 = 800$ and the SE sum = $\sqrt{4800} \times 0.372678 \approx 25.82$.

   The result (761) is a discrete value (it can only be a whole number), so the block for 761 ones runs from 760.5 to 761.5 by the methodology of pages 43 and 44.

   The value of P is the area of the tail including 761 and all numbers less than 761. By drawing a picture, it is seen that the right endpoint of the tail is at 761.5.

   For the connection to P, refer to the normal curve on page 479 and the paragraphs just above and below it.

   Then $z = \dfrac{\text{observed sum - EV sum}}{\text{SE sum}} = \dfrac{761.5 - 800}{25.82} = -1.49$. Use -1.50.

   Ignoring the continuity correction would give $(761 - 800)/25.82 = -1.51$ (use -1.50.), and hardly any difference in P.

   To get P, the area of the right tail, look up 1.50 and subtract that 86.64% from 100% to get 13.36%. Then divide by 2 to get 6.68%. Round off to 7%.

   This looks like chance variation. That somewhat large significance level P (more than 5%) makes an explanation of the null with chance error reasonable.

   It looks like the die is fair.