# Logic Check Homework

(due January 31)
Math 125 (Statistics) *Kovitz* Spring 2025

Here's a diagnostic homework to help determine the general level of logical reasoning in the class. Many of the skills tested will be used in the course.

1. In a certain community, 16% of the population has personal income under $25,000 per year and 47% has personal income over $75,000 per year. Find the percentage of the population with income between $25,000 and $75,000 per year.

2. Among the residents between 18 and 22 years old in a certain third-world country, 11% have 0 years of schooling, 12% have from 1 to 6 years of schooling, and 9% have more than 12 years of schooling. What percent of the population has 7 to 12 years of schooling? (Assume that the years of schooling are always expressed as an integer.)

3. A list with six numbers has the property that changing the sign of every other number starting with the first item on the list does not alter the average. What would happen to the average if you instead changed the sign of every other number starting with the second item on the list?

   Hint: find a list of six numbers with that property and investigate.

4. Which of the following are true for every possible list of numbers?

   Hint: consider as an example the list: 2, 2, 4, 5, 12.

   (a) Half of the items on a list are above average.

   (b) For any list, if you take the total of those entries that are below average and take the total of those entries that are above average and then compute the average of the two totals, the result will be the average of the list.

   (c) Assume that you have a list with some entries not equal to the average of the list. If you take all the entries that are not equal to the average and average them, the result will equal the average of the original list.

5. A health club in New York City with 200 members has 140 women members, 70 Italian members, and 15 Korean members.
   (In each part answer with the smallest possible, definitely-correct range.)

   (a) The number of Italian women members is surely between which two numbers?

   (b) The number of members who are either women or Italian or both is surely between which two numbers?

   (c) The number of Korean women members is surely between which two numbers?

**TURN OVER**

For Problems 6–8.

At a party the only alcoholic drinks served were wine and martinis.

6. Write an equation relating the number of people drinking alcohol ($D$) to the number having wine, the number having martini(s), and the number having both. The number drinking alcohol ($D$) is the left side of that equation. Use the letters $D$, $W$, $M$, and $B$. The equation will start $D=$. Then assume that 60 had wine, 40 had martinis, and 20 had both. Show that your equation—when filled in—gives the correct answer for the total number of people who were drinking.

7. Find, if possible, an equation for the number of people at the party who did not drink alcohol ($N$) in terms of the four letters listed above. With the numbers given in problem 6, can you find the number of people at the party who did not drink? If so, what is that number?

8. Find an expression for the number of people at the party who didn't drink in terms of the total number of people at the party and one of the four letters from Problem 6. Use $N$, $T$, and one of: $D$, $W$, $M$, $B$. Assume 150 people at the party and the data from Problem 6; from them find the number who didn't drink alcohol.

9. In a certain club, 15 members speak Spanish, 8 members speak French, and 20 members speak either French or Spanish. How many speak both languages?

10. In a certain club, 16 members speak Spanish, 4 members speak Creole, 2 members speak both Creole and Spanish, and 50 members speak neither Creole nor Spanish. Find the number of members in this club.

11. If it is not true that at least two students in a math class of twenty-three are also taking German, what could be said about the number of students in that class that are also taking German?
(Which of the following statements must be true?)

    (a) None of the students are also taking German.
    (b) One of the students is also taking German.
    (c) At most one student is also taking German.
    (d) The number of students in the class that are not also taking German must be twenty-one or less.

12. Given that it is not true that all the students in a math class are Irish, which of the following statements follow logically from that fact?

    (a) At least one of the students is not Irish.
    (b) At least two of the students are not Irish.
    (c) None of the students are Irish.
    (d) Some of the students are Irish.

**For Problems 13–15.**

A lottery game is offered in which you purchase a ticket for \$1 and designate a two-digit number between 00 and 99. A number is then selected at random.

In each problem, choose the better (that is, more advantageous to you) option or state that they are equivalent. Explain briefly.

13. (a) You win \$3 if both digits of your number match the corresponding digit of the random number.

    (b) You win \$3 if at least one of the two digits of your number matches the corresponding digits of the random number.

14. (a) You win \$100 if both digits of your number match those of the random number in exact order.

    (b) You win \$100 if both digits of your number match those of the random number in any order. For example: you picked 27 and the random number was 72; you still win.

        i. In this option, which of the following would be the better choice for the designated number: 55 or 29?

15. (a) You win \$8 if the second digit of your number matches the second digit of the random number.

    (b) You win \$8 if your two-digit number exactly matches the random number.

16. A Dean at UMass/Boston decides to send a letter to those students in his college who have a grade-point average within 0.3 of 2.0 and to those students who have a grade-point average within 0.5 of 3.2.

    Where will the grade-point of averages of his students who will not get the letter fall? Assume that grade-point averages range from 0.0 to 4.0.

**TURN OVER**

17. A company sells socks at a discount if the colors of the socks are not specified by the buyer, but are to be whatever the store chooses. The socks can be blue, black, or brown.

    How many socks must one purchase to make sure of getting a matching pair?

18. A store has in stock 10 black socks, 10 white socks, and 10 brown socks. They will fill a mail-order request with socks selected at random, of colors not specified by the buyer. How many socks must one purchase to make sure of getting at least one of each of the three colors?

19. What is the opposite (negation) of the assertion "none of my students play polo"?

20. Which of the following is the negation of "A is less than B"?

    (Which of the following describes the case when the statement is not true?)

    (a) B is less than A.
    (b) A is greater than or equal to B.
    (c) B is greater than or equal to A.
    (d) A is not equal to B.
    (e) B is not less than A.