Logic Check Homework

(due September 9)
Math 125 (Statistics) *Kovitz* Fall 2024

Here's a diagnostic homework to help determine the general level of logical reasoning in the class. Many of the skills tested will be used in the course.

1. In a certain community, 13% of the population has personal income under $25,000 per year and 44% has personal income over $75,000 per year. Find the percentage of the population with income between $25,000 and $75,000 per year.

2. Among the residents between 18 and 22 years old in a certain third-world country, 11% have 0 years of schooling, 21% have from 1 to 6 years of schooling, and 9% have more than 12 years of schooling. What percent of the population has 7 to 12 years of schooling? (Assume that the years of schooling are always expressed as an integer.)

3. A list with six numbers has the property that changing the sign of every other number starting with the first item on the list does not alter the average. What would happen to the average if you instead changed the sign of every other number starting with the second item on the list?

   Hint: find a list of six numbers with that property and investigate.

4. Which of the following are true for every possible list of numbers?

   Hint: consider as an example the list: 2, 2, 4, 5, 12.

   (a) Half of the items on a list are above average.

   (b) For any list, if you take the total of those entries that are below average and take the total of those entries that are above average and then compute the average of the two totals, the result will be the average of the list.

   (c) Assume that you have a list with some entries not equal to the average of the list. If you take all the entries that are not equal to the average and average them, the result will equal the average of the original list.

5. A health club in New York City with 200 members has 160 women members, 60 Italian members, and 10 Korean members.
   (In each part answer with the smallest possible, definitely-correct range.)

   (a) The number of Italian women members is surely between which two numbers?

   (b) The number of members who are either women or Italian or both is surely between which two numbers?

   (c) The number of Korean women members is surely between which two numbers?

TURN OVER

For Problems 6–8.

At a party the only alcoholic drinks served were wine and martinis.

6. Write an equation relating the number of people drinking alcohol (D) to the number having wine, the number having martini(s), and the number having both. The number drinking alcohol (D) is the left side of that equation. Use the letters D, W, M, and B. The equation will start $D=$. Then assume that 50 had wine, 30 had martinis, and 10 had both. Show that your equation—when filled in—gives the correct answer for the total number of people who were drinking.

7. Find, if possible, an equation for the number of people at the party who did not drink alcohol (N) in terms of the four letters listed above. With the numbers given in problem 6, can you find the number of people at the party who did not drink? If so, what is that number?

8. Find an expression for the number of people at the party who didn't drink in terms of the total number of people at the party and one of the four letters from Problem 6. Use N, T, and one of: D, W, M, B. Assume 130 people at the party and the data from Problem 6; from them find the number who didn't drink alcohol.

9. In a certain club, 16 members speak Spanish, 11 members speak French, and 20 members speak either French or Spanish. How many speak both languages?

10. In a certain club, 18 members speak Spanish, 6 members speak Creole, 4 members speak both Creole and Spanish, and 54 members speak neither Creole nor Spanish. Find the number of members in this club.

11. If it is not true that at least two students in a math class of twenty-three are also taking German, what could be said about the number of students in that class that are also taking German?
(Which of the following statements must be true?)

    (a) None of the students are also taking German.

    (b) One of the students is also taking German.

    (c) The number of students in the class that are not also taking German must be twenty-one or less.

    (d) At most one student is also taking German.

12. Given that it is not true that all the students in a math class are Irish, which of the following statements follow logically from that fact?

    (a) None of the students are Irish.

    (b) Some of the students are Irish.

    (c) At least one of the students is not Irish.

    (d) At least two of the students are not Irish.

13. One and only one of five statements referring to Barak Obama is true. Assume that he either likes or dislikes any particular vegetable. Using logical consistency, infer which of the five must be true.

    The statements are:

    (a) President Obama likes broccoli, spinach, and beets.

    (b) President Obama likes both broccoli and spinach.

    (c) President Obama likes beets.

    (d) President Obama likes asparagus.

    (e) As regards beets and asparagus, President Obama likes one and dislikes the other.

    Which is the true statement? Which of these vegetables does our former Chief Executive dislike? (Use deductive reasoning to decide.)

For Problems 14–16.

A lottery game is offered in which you purchase a ticket for $1 and designate a two-digit number between 00 and 99. A number is then selected at random.

In each problem, choose the better (that is, more advantageous to you) option or state that they are equivalent. Explain briefly.

14. (a) You win $3 if at least one of the two digits of your number matches the corresponding digit of the random number.

    (b) You win $3 if both digits of your number match the corresponding digits of the random number.

15. (a) You win $100 if both digits of your number match those of the random number in exact order.

    (b) You win $100 if both digits of your number match those of the random number in any order. For example: you picked 27 and the random number was 72; you still win.

    In this option, which of the following would be the better choice for the designated number: 55 or 29?

16. (a) You win $8 if your two-digit number exactly matches the random number.

    (b) You win $8 if the second digit of your number matches the second digit of the random number.

17. A Dean at UMass/Boston decides to send a letter to those students in his college who have a grade-point average within 0.2 of 2.0 and to those students who have a grade-point average within 0.1 of 3.2.

    Where will the grade-point of averages of his students who will not get the letter fall? Assume that grade-point averages range from 0.0 to 4.0.

TURN OVER

18. You have 4 black socks, 4 white socks, 4 blue socks, and 4 red socks in a drawer. Assuming that it is too dark to see and all socks feel the same, how many socks must you pick to make sure you get a matching pair?

19. You have 5 black socks, 5 white socks, 5 blue socks, and 5 red socks in a drawer. Assuming that it is too dark to see and all socks feel the same, how many socks must you pick to make sure that you get four different colors?

20. What is the opposite (negation) of the assertion "at least one of my students speaks Hungarian"?