Practice for Test 1

Math 130 *Kovitz* Spring 2019

The test is on Tuesday, March 5.

Problems 1 through 52: True or false.

1. A line segment with endpoints at $(-2, 9)$ and $(7, -1)$ will have a midpoint at $(3.5, 5)$.

2. If the coordinates of A are $(-1, 4)$ and the coordinates of the midpoint of $\overline{AB}$ are $(-3, 6)$, the coordinates of B are $(-5, 8)$ and the length of the line segment AB is $4\sqrt{2}$.

3. The length of the line segment connecting the points whose coordinates are $(-1, -2)$ and $(1.5, 4)$ is exactly 6.5.

4. The straight line through $(5, -3)$ and $(8, 12)$ has a slope of 3.

5. The graph of $3y - x = b$ is parallel to the graph of $y = -\frac{1}{3}x + b$.

6. The graphs of $\frac{1}{2}y + x = 3$ and $y = 2x - 6$ are perpendicular and have the same x-intercept.

7. An equation of the line that is parallel to $y = -3x + 6$ and passes through the point $(0, -3)$ is $y = 3x - 3$.

8. The line with slope $= \frac{3}{2}$ through the point $(5, -2)$ will also contain the point $(8, 0)$.

9. Given points $A(0, 0)$, $B(2, 1)$, and $C(1, 2)$, none of the lines AB, AC, or BC are parallel or perpendicular; but the lengths of lines AB and AC are equal.

10. An equation of the line that is parallel to $y + 3x + 4 = 0$ and has the same y-intercept as $y = -5x - 7$ is $y = 3x - 7$.

11. For the points $A(4, -3)$ and $B(2, 3)$, an equation of the line that is perpendicular to $\overline{AB}$ at its midpoint is $y = \frac{1}{3}x - 1$.

12. The equation $y = \sqrt{x+1}\sqrt{x-1}$ has domain $[1, \infty)$, is an increasing function with x-intercept at $(1, 0)$, and the rest of its points lie only in the first quadrant.

13. The equation $y = \pm\sqrt{1 - x^2}$ has domain $[-1, 1]$, is not a function, and graphs as the unit circle.

14. The equation $y^2 = 25 - x^2$ has domain $[-5, 5]$, is not a function, and graphs as a circle with radius 5 units that is centered at the origin.

15. The equation $y = \sqrt{9 - x^2}$ is an even function that has domain $[-3, 3]$, points in the first and second quadrants, and graphs as an upper semicircle with radius 3 units that is centered at the origin.

16. The graph of the equation $y^2 = \dfrac{24}{x}$ is located in the first and second quadrants, has no intercepts, and has asymptotes on the x- and y-axes.

17. The graph of the equation $3x + 5y + 8 = 0$ is a straight line with points located in all four quadrants.

18. The graph of the equation $x^4 = y^4$ consists of the two straight lines $y = x$ and $y = -x$ with intercepts at the origin and (altogether) points in all four quadrants.

19. The graph of the equation $\sqrt{x} \cdot \sqrt{y} = 1$ is the portion of the graph of $y = 1/x$ that is located in the first quadrant, and it has vertical and horizontal asymptotes but no intercepts.

20. The graph of $y = \sqrt{x+3} + 3$ is the same as the graph of the square root function $y = \sqrt{x}$, except that it was shifted 3 units to the right and 3 units up.

21. The equation $y = \pm x$ does not describe a function of x, but the equation $y^2 = x$ does describe a function of x.

22. The equation $|y| = x$ describes y as a function of x.

23. In the expression $f(7) = 11$, the number 7 is a value of x (an input to the function), and there is no other real number a with $f(7) = a$.

24. If $f(7) = 11$ and $f(4) = 11$, f cannot be a function.

25. The fn. $f(x) = \sqrt{x+13} - 17$ has domain $[13, \infty)$ and range $[-17, \infty)$.

26. The function $g(x) = \dfrac{x}{|x|}$ has domain all real numbers except zero and range all real numbers except zero.

27. The functions $f(x) = \dfrac{\sqrt{x+4}}{\sqrt{x+3}}$ and $g(x) = \dfrac{\sqrt{x+3}}{\sqrt{x+4}}$ have the same domain: $[-3, \infty)$.

28. The domain of the function $f(x) = \dfrac{9}{(x-9)^2 - 9}$ is all real numbers except 12.

29. When $f(x) = x^2 + x$, the expression $\dfrac{f(x+3) - f(x)}{3}$ simplifies to $2x + 4$.

30. When $f(x) = 2x^2 + 5x - 3$ and $h \neq 0$, the expression $\dfrac{f(x+h) - f(x)}{h}$ simplifies to $4x + 2h + 5$.

31. The graph of $x^2 + xy + y^2 = 10$ has symmetry across the x-axis and symmetry across the y-axis.

32. The unit circle and the graph of the equation $|y| = |x|$ each have all four unsual symmetries (x-axis, y-axis, origin, and $y = x$).

33. The graph of the equation $y = |x|$ has symmetry across the x-axis.

34. The graph of the equation $y = x^2$ is symmetric across the x-axis.

35. The graph of the equation $y = \dfrac{1}{x}$ is symmetric through the origin and across the line $y = x$.

36. The graph of the equation $y = 2x + 7$ is the reflection through the origin of the graph of $y = 2x - 7$. (Not sure? Try a few points on the first graph and reflect them through the origin and check if they are on the second graph.)

37. The graph of $y = \dfrac{1}{\sqrt{-x}}$ is the reflection across the x-axis of the graph of $y = \dfrac{1}{\sqrt{x}}$.

38. The graph of the function $f(x) = \sqrt{x-5} + 7$ is just the graph of the parent function $f(x) = \sqrt{x}$ moved 5 to the right and up 7.

39. The graph of the function $g(x) = |x+1| + 1$ is just the graph of the parent function $g(x) = |x|$ moved 1 to the right and up 1.

40. The graph of the function $h(x) = (x+6)^3 - 11$ is just the graph of the parent function $h(x) = x^3$ moved 6 to the left and down 11.

41. The function $y = \sqrt{(x^2-1)x}$ is odd.

42. The equation $y = \dfrac{x^3 - x}{1 - 2x^2}$ defines an odd function.

43. The function $y = |1 - x^3|$ is odd.

44. The function $f(x) = \dfrac{2}{x}$ is odd.

45. The function $f(x) = -1$ is odd.

46. The equation $y = \sqrt{x^2-1}\sqrt{x^2+1}$ defines an even function.

47. The equation $y = \dfrac{1}{x} + \dfrac{x}{1-x^2}$ defines an odd function.

48. The function $f(x) = 0$ is both even and odd.

49. The function $y = \dfrac{\sqrt{x}}{\sqrt{x^2-1}}$ is an odd function.

50. The function $y = |x^3 - x|$ is odd.

51. The function $y = \dfrac{1}{x^2} + x$ is neither even nor odd.

52. The function $y = \left(\sqrt{x}\right)^4$ is neither even nor odd (if we consider functions only over the real numbers).

Answers follow on subsequent pages.

Answers.

1. False; the midpoint will be at $(2.5, 4)$.
2. True.
3. True, because $2.5^2 + 6^2 = 6.5^2$.
4. False. The slope is 5.
5. False. It is parallel to the graph of $y = \frac{1}{3}x + b$.
6. False. The x-intercepts are the same, but the slopes are -2 and 2. Those slopes are not negative reciprocals.
7. False. A correct equation would be $y = -3x - 3$.
8. False. A new point could be found by going up 3 and right 2; that produces the point $(7, 1)$. The point $(8, 0)$ was determined by going right 3 and up 2; that does not correspond to a slope of $\frac{3}{2}$. The slope to the point $(8, 0)$ is $\frac{2}{3}$. The slope is the change in y over the change in x.
9. True.
10. False. An equation could be $y = -3x - 7$, as the original equation had slope of -3.
11. True.
12. True.
13. True.
14. True.
15. True.
16. False. It is located in the first and fourth quadrants.
17. False. There are no points in the first quadrant; and a straight line can never pass through all four quadrants.
18. True.
19. True.
20. False. It was shifted 3 units to the left and up 3.
21. False. Neither equation describes a function of x; both graphs fail the Vertical Line Test.
22. False. For example, the points $(3, 3)$ and $(3, -3)$ are both on the graph, violating the Vertical Line Test.
23. True.
24. False. It is perfectly all right for a two different inputs to have the same output. This merely means that the function is not one-to-one.
25. False. The range is correct, but the domain is $[-13, \infty)$.
26. False. The domain is correct, but the range is just the two real numbers -1 and 1.

27. False. The listed domain is correct for g, but f has domain $(-3, \infty)$ because -3 is not in the domain.

28. False. The domain is all real numbers except 6 and 12.

29. True.

30. True.

31. False. It has symmetry through the origin and across the line $y = x$.

    But when you reflect across the x- or y-axis, the equation of the reflection comes out to be $x^2 - xy + y^2 = 0$. That equation is not equivalent to the original.

    For the origin or the line $y = x$, the equation of the reflection comes up to be exactly the same as the original.

32. True.

33. False. It has symmetry across the y-axis.

34. False. It has symmetry across the y-axis.

35. True.

36. True.

37. False. It is the reflection across the y-axis.

38. True.

39. False. It was moved 1 to the left.

40. True.

41. False.

    The domain is unbalanced. For example 2 is in the domain, but -2 is not.

    The actual domain turns out to be $-1 \leq x \leq 0$ or $x \geq 1$. That domain is not balanced.

42. True.

43. False. For example $f(2) = 7$, but $f(-2) = 9$.

44. True.

45. False. It is an even function.
    Note that $f(a) = -1$ and $f(-a) = -1$ for all a, making $f(-a) = f(a)$.

46. True. Note that the domain $x \leq -1$ or $x \geq 1$ is a balanced domain, which supports the finding.

47. True. Because $f(a) = \dfrac{1}{a} + \dfrac{a}{1-a^2}$ and $f(-a) = \dfrac{1}{-a} + \dfrac{-a}{1-a^2} = -f(a)$.

48. True. Because $f(a) = 0 = f(-a)$ and $-f(a) = -0 = 0 = f(-a)$.

49. False. The domain is not balanced. For example 2 is in the domain, but -2 is not in the domain. Or just observe that the domain contains no negative values.

50. False. It is even. $f(a) = |a| \cdot |a^2 - 1| = |-a| \cdot |(-a)^2 - 1| = f(-a)$.

51. True.

52. True. The domain is $[0, \infty)$, which is unbalanced.