Practice for Test 2

Math 130 *Kovitz* Spring 2019

The test is on Tuesday, April 16.

Problems 1 through 71: True or false.

1. The standard form of the quadratic equation $y = 2x^2 - 32x + 100$ is $y = 2(x-8)^2 + 36$.

2. The standard form of the quadratic equation $y = 2x^2 - 16x + 100$ is $y = 2(x-4)^2 + 68$.

3. An alternate, equivalent form of the quadratic equation $y = x^2 + x + 1$ is $y = (x+1)^2 - x$, but it is not in standard form.

4. The standard form of the quadratic equation $y = 4x^2 + 2x + 1$ is $y = 4(x + \frac{1}{2})^2$.

5. The graph of the quadratic equation $y = 4x^2 + 4x + 1$ has y-intercept at $(0,1)$, x-intercept at $\left(\frac{1}{2}, 0\right)$, and vertex at $\left(-\frac{1}{2}, 0\right)$.

6. The graph of the equation $y = 7x^2 + 4x + 1$ is a parabola with vertex at the point $(-2/7,\ 3/7)$.

7. By factoring the equation $y = x^2 - 10x + 21$, it is easy to see that its x-intercepts are $(3,0)$ and $(7,0)$ and that its line of symmetry is $x = 5$.

8. The standard form of the equation $y = 8x^2 - 8x + 2$ is $y = 8(x - \frac{1}{2})^2$, and the graph has y-intercept at $(0,2)$ but only one x-intercept at $\left(\frac{1}{2}, 0\right)$.

9. The x-intercepts of the graph of $y = \frac{1}{2}(x-3)^2 - 8$ are $(1,0)$ and $(5,0)$.

10. The graph of $y = x^2 + 2x + 2$ has x-intercepts at $(-1,0)$ and $(-2,0)$.

11. The graph of $y = -x^2 - x - 1$ has no x-intercepts.

12. The graphs of $y = x^2 + 11x + 8$ and $y = -x^2 + 11x - 8$ have the same x-intercepts.

13. The graph of $y = x^2 + 2.75x + 2$ has no x-intercepts.

14. The graphs of $y = x^2 + 11x + 8$ and $y = -x^2 - 11x - 8$ have the same x-intercepts.

15. The graphs of two quadratic functions in the form $y = ax^2 + bx + c$ with the same value of a will have the same shape.

16. If a quadratic function whose graph opens up has its vertex above the x-axis, there will be no x-intercepts.

17. The inverse function of $f(x) = \dfrac{3x+1}{5x-2}$ is $g(x) = \dfrac{2x+1}{5x-3}$.

18. The inverse of the function $f(x) = \dfrac{(x-7)^3 - 5}{2}$ is the function $g(x) = 2\sqrt[3]{x+7} + 5$.

19. The inverse of the function $f(x) = \dfrac{(x-7)^3 - 5}{2}$ is the function $g(x) = 2\sqrt[3]{x+7} + 10$.

20. The functions $f(x) = \dfrac{2x-7}{3}$ and the function $g(x) = \dfrac{3x+7}{2}$ are inverses of each other.

21. The functions $f(x) = \dfrac{2x-5}{3}$ and the function $g(x) = \dfrac{3}{2x-5}$ are inverses of each other.

22. The inverse of the function $y = x^3 - 1$ is the function $y = \sqrt[3]{x+1}$.

23. The function $y = |x|$ does not have an inverse function.

24. The inverse function of $f(x) = \dfrac{1}{x+1}$ is $g(x) = \dfrac{1-x}{x}$.

25. A function that is symmetric across the line $y = x$ must pass the horizontal line test and must be its own inverse.

26. A one-to-one function that is symmetric across the line $y = x$ is its own inverse.

27. The graph of $y = 2^x$ passes the Horizontal Line Test, lies in the first and second quadrants, has a y-intercept at $(0,1)$, and has an asymptote on the negative x-axis.

28. On the graph of $y = 2^x$, if the 2nd coordinate of point B is the square of the first coordinate of point A, the first coordinate of point B must be twice the first coordinate of point A.

29. On the graph of $y = 2^x$, if the 1st coordinate of point B is 5 more than the 1st coordinate of point A, the 2nd coordinate of point B must be 5 times the 2nd coordinate of point A.

30. The function $y = (1/2)^x$ is decreasing.

31. The function $y = \log_2 x$ is a decreasing function whose graph lies in the first and second quadrants and has an x-intercept of $(1,0)$.

32. The graph of $y = \log_2 x$ has its asymptote on the negative y-axis.

33. The equation $y = \log_2 x$ defines a one-to-one function.

34. The function $y = \log_2 x$ has domain all positive reals and range all reals.

35. The value of $\log_3 41$ is not between 27 and 81, but it is between $\log_3 27$ and $\log_3 81$.

36. The value of $\log_2(1/17)$ is between $1/32$ and $1/16$.

37. The value of $\log_2(1/17)$ is between -5 and -4.

38. The value of $\log_2(-5)$ is between -3 and -2.

39. The value of $\log_8(5/6)$ is between -1 and 0.

40. Since the functions $f(x) = \log_a x$ and $g(x) = a^x$ (with $a > 0$ and $a \neq 1$) are inverses of each other, it is surely true that $\log_a(a^p) = a^{\log_a p}$, assuming that p is positive.

41. Since the functions $f(x) = \log_4 x$ and $g(x) = 4^x$ are inverses of each other, $4^{\log_4 \frac{1}{64}} = -3$.

42. Since the functions $f(x) = \log_3 x$ and $g(x) = 3^x$ are inverses of each other, $\log_3(3^7) = 81$.

43. Since the functions $f(x) = \log_3 x$ and $g(x) = 3^x$ are inverses of each other, $\log_3(3^0) = 1$.

44. When it is rewritten in exponential form, $\log b = c$ becomes $10^b = c$.

45. The inverse of the function $y = 10^x$ is the common logarithm function with equation $y = \log_{10} x$.

46. When it is rewritten in exponential form, $\log_9 2187 = 3.5$ becomes $9^{3.5} = 2187$.

47. In exponential form, $\log_2(32\sqrt{2}) = 5.5$ becomes $2^{5.5} = 32\sqrt{2}$.

48. In logarithm form, $4^{3/4} = 2\sqrt{2}$ becomes $\log_4\left(\frac{3}{4}\right) = 2\sqrt{2}$.

49. The difference of the logarithms of two positive numbers is equal to the logarithm of the quotient of the two numbers.

50. The logarithm of the fifth root of a positive real number is equal to 1/5 of the logarithm of the number.

51. The logarithm of the square of a positive real number is equal to the square of the logarithm of the number.

52. $\log\left(\dfrac{1}{ab}\right) = -(\log a + \log b)$.

53. $\log 2.4 = \log 2 \times \log 0.4$.

54. $\log 2.4 = \log 2 + \log 0.4$.

55. $\log_2\left(\frac{1/8}{16}\right) = \log_2(1/8) - \log_2 16 = -3 - 4 = -7$.

56. $\log_4\left(13^{1/4}\right) = -1 \times \log_4 13$.

57. When $\log_b 2 = 1.637$ and $\log_b 2.72 = 2.363$, $\log_b 5.44 = 4$.

58. If $\log_b 2 = 1.637$ and $\log_b 2.72 = 2.363$, $\log_b 8 = 8 \times 1.637 = 13.096$.

59. When $\log_b 2 = 1.637$ and $\log_b 2.72 = 2.363$, $\log_b 8 = 3 \times 1.637 = 4.911$.

60. The equation $\log_2(x-1) - \log_2 x = 1$ has no real-numbered solutions.

61. The equation $\log_3 x - \log_3(x-1) = 2$ has only one real-numbered solution: $x = 1.125$.

62. The equation $\log(2x) - \log(x-2) = 1$ doesn't have an integer solution.

63. The equation $\log(x) - \log(x-99) = 1$ has only one real-numbered solution, $x = 100$.

64. The equation $\dfrac{\log_2(5x-1)}{\log_2 2x} = 1$ has only one solution $x = 1/3$, while the equation $\log_2(5x-1) - \log_2 2x = 1$ has no solutions.

65. Change of base proves that $\log_{\sqrt{a}} b = \dfrac{1}{2}\log_a b$ in all cases.

66. It is always true that $\dfrac{1}{\log_b a} = \log_a b$, and it can be proved by changing the base of $\log_a b$ to base b.

67. $\log_b(-a) = -\log_b a$ in all cases.

68. Based on the approximation $\log 2 = 0.3$, $\log \sqrt[3]{2}$ is approximately equal to $\log \sqrt[10]{10}$.

69. Based on the approximation $\log 2 = 0.301$, an approximation for $\log_{64} 5$ is about $699/1806$, which makes some sense since $\log_{64} 5$ was estimated to be between $1/3$ and $1/2$.

    Hint: Start with $\log_{64} 5$ and change the base to base 10. Then rewrite 64 as a power of 2 and further simplify to get an approximate fraction.

70. Using change of base, the value of $\log_8 \sqrt[4]{2}$ comes out to be the exact fraction $1/12$.

71. Using change of base, $\log_{32} 2^{0.32}$ is shown to be exactly equal to 0.01.

Answers follow on subsequent pages.

Answers.

1. False. It is $y = 2(x-8)^2 - 28$.

2. True.

3. True.

4. False. The correct standard form is $y = 4\left(x + \frac{1}{4}\right)^2 + \frac{3}{4}$.

5. False. The x-intercept is at the same point as the vertex: $\left(-\frac{1}{2}, 0\right)$.

6. True.

7. True. It factors as $y = (x-3)(x-7)$.

8. True.

9. False. They are at $(-1,0)$ and $(7,0)$.

10. False. This graph has no x-intercepts; the graph of $y = x^2 + 3x + 2$ has those two x-intercepts.

11. True.

12. False. The x-intercepts of $x^2 + 11x + 8$ are both negative, while the x-intercepts of $-x^2 + 11x - 8$ are both positive, the the additive inverses of the first two.

    There are two rather simple ways to establish this.

    First, note that–because of the result of substiting $(-x)$ for x and $(-y)$ for y–the graphs of $y = x^2 + 11x + 8$ and $y = -x^2 + 11x - 8$ must be reflections of each other through the origin.

    Looking at the coordinates of specific points, a point on the first graph reflected through the origin must be on the second graph. The x-intercepts of the graph of $y = x^2 + 11x + 8$ are of the form $(a, 0)$. A point is reflected through the origin by changing the sign of both coordinates. So the point $(-a, 0)$ will be an x-intercept of the graph of $y = -x^2 + 11x - 8$. Because the line of symmetry of $y = x^2 + 11x + 8$ is clearly not the y-axis, its two x-intercepts cannot be $\pm a$.

    That makes the statement of the problem false.

    Reflecting through the origin by putting a pin in the origin and rotating 180 degrees, the x-interecpts on the positive x-axis end up on the negative x-axis and the x-intercepts on the negative x-axis end up on the positive x-axis. Unless the original intercept was only the point $(0,0)$ or the original two x-intercepts were $(-a, 0)$ and $(a, 0)$, the resulting points will not be the same. Because the line of symmetry in this case is not the y-axis, the original x-intercepts were not $(\pm a, 0)$.

    Again, that makes the statement of the problem false.

13. True.

14. True. It is easy to see since each is the reflection of the other across the x-axis and two graphs reflected across the x-axis must have the same x-intercepts since points on the x-axis remain unchanged under a reflection across the x-axis.

15. True. One is just a translation of the other. That is clear when both are expressed in standard form.

16. True.

17. True.

18. False. The inverse is $\sqrt[3]{2x+5}+7$.

19. False. The inverse is $\sqrt[3]{2x+5}+7$.

20. True.

21. False. These two functions are reciprocals of each other.
The inverse of f is the function $f^{-1}(x) = \dfrac{3x+5}{2}$.

22. True.

23. True. It is not one-to-one; it fails the Horizontal Line Test.

24. True. Note that the function $g(x)$ has an alternate form $\dfrac{1}{x} - 1$.

25. True.

26. True.

27. True.

28. True.

29. False. It is 32 times as much.

30. True.

31. False. It is an increasing function which lies in the first and fourth quadrants.

32. True.

33. True.

34. True.

35. True.

36. False. It is between $\log_2(1/32)$ and $\log_2(1/16)$.

37. True.

38. False. The logarithms of negative numbers are not defined.

39. True.

40. True.

41. False. It is equal to $\frac{1}{64}$.

42. False. It is equal to 7.

43. False. It is equal to 0.

44. False. It becomes $10^c = b$.

45. True.

46. True.

47. True.

48. False. It becomes $\log_4(2\sqrt{2}) = \frac{3}{4}$.

49. True.

50. True.

51. False. It is equal to twice the logarithm of the number.

52. True.

53. False. The other way around.
    $\log(2 \times 0.4) = \log 2 + \log 0.4$.

54. False. $\log(2 \times 0.4) = \log 2 + \log 0.4$.
    2×0.4 is 0.8 not 2.4.

55. True.

56. False. It equals $(1/4) \times \log_4 13$.

57. True.

58. False. The value 8 is 2 cubed, so the answer is triple the log of 2.

59. True.

60. True.

61. True.

62. True. The unique solution is $x = 2.5$, but 2.5 is not an integer.

63. False. The one solution is $x = 110$. If the expression on the left were set equal to 2 instead of 1, 100 would be the correct solution.

64. False. The given solution is correct for the first equation but the second equation has the solution $x = 1$.

65. False. It proves that $\log_{\sqrt{a}} b = 2\log_a b$.

66. True.

67. False. The logarithms of a and $-a$ cannot both be defined as only one of the numbers a and $-a$ could ever be positive.

68. True. One third of 0.3 is equal to one tenth of 1.

69. True. Note: Just after the change-of-base step, it was helpful—in the process of finding $\log 5$—to replace 5 by 10/2 and take the log of that fraction.

70. True.

71. False. It is shown to be exactly equal to 0.064.