Practice for Test 4

Math 130 *Kovitz* Fall 2018
The test is on Tuesday, October 30.

Problems 1 through 71: True or false.

1. The equation $y = \sqrt{x+1}\sqrt{x-1}$ has domain $[1, \infty)$, is an increasing function with x-intercept at $(1, 0)$, and the rest of its points lie only in the first quadrant.

2. The equation $y = \pm\sqrt{1-x^2}$ has domain $[-1, 1]$, is not a function, and graphs as the unit circle.

3. The equation $y^2 = 25 - x^2$ has domain $[-5, 5]$, is not a function, and graphs as a circle with radius 5 units that is centered at the origin.

4. The equation $y = \sqrt{9-x^2}$ is an even function that has domain $[-3, 3]$, points in the first and second quadrants, and graphs as an upper semicircle with radius 3 units that is centered at the origin.

5. The graph of the equation $y^2 = \dfrac{24}{x}$ is located in the first and second quadrants, has no intercepts, and has asymptotes on the x- and y-axes.

6. The graph of the equation $3x + 5y + 8 = 0$ is a straight line with points located in all four quadrants.

7. The graph of the equation $x^4 = y^4$ consists of the two straight lines $y = x$ and $y = -x$ with intercepts at the origin and (altogether) points in all four quadrants.

8. The graph of the equation $\sqrt{x} \cdot \sqrt{y} = 1$ is the portion of the graph of $y = 1/x$ that is located in the first quadrant, and it has vertical and horizontal asymptotes but no intercepts.

9. The graph of the equation $x^2 + 8x + y^2 - 9 = 0$ is a circle with center at $(-4, 0)$ and a radius of 3.

10. The equations $x^2 - 4x + y^2 = -4$ and $(x-2)^2 + y^2 = 0$ are equivalent and their graphs consist of the single point $(2, 0)$.

11. The graph of the equation $x^2 + y^2 = 0$ is not a circle, but the graph of $x^2 - 2x + y^2 = 0$ is a circle.

12. The graph of $y = \sqrt{x+3} + 3$ is the same as the graph of the square root function $y = \sqrt{x}$, except that it was shifted 3 units to the right and 3 units up.

13. The equation $y = \pm x$ does not describe a function of x, but the equation $y^2 = x$ does describe a function of x.

14. The equation $|y| = x$ describes y as a function of x.

15. In the expression $f(7) = 11$, the number 7 is a value of x (an input to the function), and there is no other real number a with $f(7) = a$.

16. If $f(7) = 11$ and $f(4) = 11$, f cannot be a function.

17. The fn. $f(x) = \sqrt{x+13} - 17$ has domain $[13, \infty)$ and range $[-17, \infty)$.

18. The function $g(x) = \dfrac{x}{|x|}$ has domain all real numbers except zero and range all real numbers except zero.

19. The functions $f(x) = \dfrac{\sqrt{x+4}}{\sqrt{x+3}}$ and $g(x) = \dfrac{\sqrt{x+3}}{\sqrt{x+4}}$ have the same domain: $[-3, \infty)$.

20. The domain of the function $f(x) = \dfrac{9}{(x-9)^2 - 9}$ is all real numbers except 12.

21. When $f(x) = x^2 + x$, the expression $\dfrac{f(x+3) - f(x)}{3}$ simplifies to $2x + 4$.

22. When $f(x) = 2x^2 + 5x - 3$ and $h \neq 0$, the expression $\dfrac{f(x+h) - f(x)}{h}$ simplifies to $4x + 2h + 5$.

23. The graph of $x^2 + xy + y^2 = 10$ has symmetry across the x-axis and symmetry across the y-axis.

24. The unit circle and the graph of the equation $|y| = |x|$ each have all four unsual symmetries (x-axis, y-axis, origin, and $y = x$).

25. The graph of the equation $y = |x|$ has symmetry across the x-axis.

26. The graph of the equation $y = x^2$ is symmetric across the x-axis.

27. The graph of the equation $y = \dfrac{1}{x}$ is symmetric through the origin and across the line $y = x$.

28. The graph of the equation $y = 2x + 7$ is the reflection through the origin of the graph of $y = 2x - 7$. (Not sure? Try a few points on the first graph and reflect them through the origin and check if they are on the second graph.)

29. The graph of $y = \dfrac{1}{\sqrt{-x}}$ is the reflection across the x-axis of the graph of $y = \dfrac{1}{\sqrt{x}}$.

30. The graph of the function $f(x) = \sqrt{x-5} + 7$ is just the graph of the parent function $f(x) = \sqrt{x}$ moved 5 to the right and up 7.

31. The graph of the function $g(x) = |x+1| + 1$ is just the graph of the parent function $g(x) = |x|$ moved 1 to the right and up 1.

32. The graph of the function $h(x) = (x+6)^3 - 11$ is just the graph of the parent function $h(x) = x^3$ moved 6 to the left and down 11.

33. The function $y = \sqrt{(x^2-1)x}$ is odd.

34. The equation $y = \dfrac{x^3 - x}{1 - 2x^2}$ defines an odd function.

35. The function $y = |1 - x^3|$ is odd.

36. The function $f(x) = \dfrac{2}{x}$ is odd.

37. The function $f(x) = -1$ is odd.

38. The equation $y = \sqrt{x^2 - 1}\sqrt{x^2 + 1}$ defines an even function.

39. The equation $y = \dfrac{1}{x} + \dfrac{x}{1 - x^2}$ defines an odd function.

40. The function $f(x) = 0$ is both even and odd.

41. The function $y = \dfrac{\sqrt{x}}{\sqrt{x^2 - 1}}$ is an odd function.

42. The function $y = |x^3 - x|$ is odd.

43. The function $y = \dfrac{1}{x^2} + x$ is neither even nor odd.

44. The function $y = \left(\sqrt{x}\right)^4$ is neither even nor odd (if we consider functions only over the real numbers).

45. The standard form of the quadratic equation $y = 2x^2 - 32x + 100$ is $y = 2(x - 8)^2 + 36$.

46. The standard form of the quadratic equation $y = 2x^2 - 16x + 100$ is $y = 2(x - 4)^2 + 68$.

47. An alternate, equivalent form of the quadratic equation $y = x^2 + x + 1$ is $y = (x + 1)^2 - x$, but it is not in standard form.

48. The standard form of the quadratic equation $y = 4x^2 + 2x + 1$ is $y = 4(x + \frac{1}{2})^2$.

49. The graph of the quadratic equation $y = 4x^2 + 4x + 1$ has y-intercept at $(0, 1)$, x-intercept at $\left(\frac{1}{2}, 0\right)$, and vertex at $\left(-\frac{1}{2}, 0\right)$.

50. The graph of the equation $y = 7x^2 + 4x + 1$ is a parabola with vertex at the point $(-2/7,\ 3/7)$.

51. By factoring the equation $y = x^2 - 10x + 21$, it is easy to see that its x-intercepts are $(3, 0)$ and $(7, 0)$ and that its line of symmetry is $x = 5$.

52. The standard form of the equation $y = 8x^2 - 8x + 2$ is $y = 8(x - \frac{1}{2})^2$, and the graph has y-intercept at $(0, 2)$ but only one x-intercept at $\left(\frac{1}{2}, 0\right)$.

53. The x-intercepts of the graph of $y = \frac{1}{2}(x - 3)^2 - 8$ are $(1, 0)$ and $(5, 0)$.

54. The graph of $y = x^2 + 2x + 2$ has x-intercepts at $(-1, 0)$ and $(-2, 0)$.

55. The graph of $y = -x^2 - x - 1$ has no x-intercepts.

56. The graphs of $y = x^2 + 11x + 8$ and $y = -x^2 + 11x - 8$ have the same x-intercepts.

57. The graph of $y = x^2 + 2.75x + 2$ has no x-intercepts.

58. The graph of $y = x^2 + \sqrt{8}x + 2$ is a perfect square with a single x-intercept.

59. The graphs of $y = x^2 + 11x + 8$ and $y = -x^2 - 11x - 8$ have the same x-intercepts.

60. The graphs of two quadratic functions in the form $y = ax^2 + bx + c$ with the same value of a will have the same shape.

61. If a quadratic function whose graph opens up has its vertex above the x-axis, there will be no x-intercepts.

62. If twice the first number plus half the second number equals 6, their largest possible product is 18.

63. The largest area that a farmer can enclose by constructing a rectangular pen from 40 feet of fencing, if he uses one wall of his barn for a wall of the pen, is 200 square feet.

64. The area of the largest rectangular field that 120 feet of fencing can enclose is 3,600 square feet.

65. When a number is multiplied by a second number, 6 bigger, and then 24 is added to that product the smallest possible result after the 24 is added is -9.

66. For a certain object thrown up from the ground, its height y above ground in meters at time t seconds when $t \geq 0$ and $y \geq 0$ can be determined by the function $y = -5t^2+10t$; in such a case the maximum height above ground will be 10 meters.

67. For the functions $f(x) = x^2 + 8x + 1$ and $g(x) = x - 4$, the composite function $(f \circ g)(x) = x^2 - 15$.

68. For the functions $f(x) = \dfrac{x+1}{2}$ and $g(x) = \dfrac{x-1}{2}$, the composite functions $(f \circ g)(x)$ and $(g \circ f)(x)$ come out the same, to $y = \dfrac{x}{4}$.

69. When $f(x) = \dfrac{x-2}{3}$, $f(f(x))$ will equal $\dfrac{x-8}{9}$.

70. For the functions $f(x) = \dfrac{2x-3}{x+1}$ and $g(x) = \dfrac{x+3}{2-x}$, the composite functions $f \circ g$ and $g \circ f$ both have the same formula: $y = x$.

71. When a current, I, flows through a given electrical circuit, the power, W, of the circuit can be determined by the function $W(I) = 100I - 10I^2$; in such a case the maximum power that can be supplied to the circuit is 500.

Answers follow on subsequent pages.

Answers.

1. True.
2. True.
3. True.
4. True.
5. False. It is located in the first and fourth quadrants.
6. False. There are no points in the first quadrant; and a straight line can never pass through all four quadrants.
7. True.
8. True.
9. False. The radius is 5.
10. True.
11. True. The graph of $x^2 + y^2 = 0$ is the origin only, the other is a circle centered at $(1, 0)$ with radius 1.
12. False. It was shifted 3 units to the left and up 3.
13. False. Neither equation describes a function of x; both graphs fail the Vertical Line Test.
14. False. For example the points $(3, 3)$ and $(3, -3)$ are both on the graph, violating the Verical Line Test.
15. True.
16. False. It is perfectly alright for a two different inputs to have the same output. This merely means that the function is not one-to-one.
17. False. The range is correct, but the domain is $[-13, \infty)$.
18. False. The domain is correct, but the range is just the two real numbers -1 and 1.
19. False. The listed domain is correct for g, but f has domain $(-3, \infty)$ because -3 is not in the domain.
20. False. The domain is all real numbers except 6 and 12.
21. True.
22. True.
23. False. It has symmetry through the origin and across the line $y = x$.

    But when you reflect across the x- or y-axis, the equation of the reflection comes out to be $x^2 - xy + y^2 = 0$. That equation is not equivalent to the original.

    For the origin or the line $y = x$, the equation of the reflection comes up to be exactly the same as the original.

24. True.

25. False. It has symmetry across the y-axis.

26. False. It has symmetry across the y-axis.

27. True.

28. True.

29. False. It is the reflection across the y-axis.

30. True.

31. False. It was moved 1 to the left.

32. True.

33. False.

    The domain is unbalanced. For example 2 is in the domain, but -2 is not.

    The actual domain turns out to be $-1 \leq x \leq 0$ or $x \geq 1$. That domain is not balanced.

34. True.

35. False. For example $f(2) = 7$, but $f(-2) = 9$.

36. True.

37. False. It is an even function.

    Note that $f(a) = -1$ and $f(-a) = -1$ for all a, making $f(-a) = f(a)$.

38. True. Note that the domain $x \leq -1$ or $x \geq 1$ is a balanced domain, which supports the finding.

39. True. Because $f(a) = \dfrac{1}{a} + \dfrac{a}{1-a^2}$ and $f(-a) = \dfrac{1}{-a} + \dfrac{-a}{1-a^2} = -f(a)$.

40. True. Because $f(a) = 0 = f(-a)$ and $-f(a) = -0 = 0 = f(-a)$.

41. False. The domain is not balanced. For example 2 is in the domain, but -2 is not in the domain. Or just observe that the domain contains no negative values.

42. False. It is even. $f(a) = |a| \cdot |a^2 - 1| = |-a| \cdot |(-a)^2 - 1| = f(-a)$.

43. True.

44. True. The domain is $[0, \infty)$, which is unbalanced.

45. False. It is $y = 2(x-8)^2 - 28$.

46. True.

47. True.

48. False. The correct standard form is $y = 4\left(x + \frac{1}{4}\right)^2 + \frac{3}{4}$.

49. False. The x-intercept is at the same point as the vertex: $\left(-\frac{1}{2}, 0\right)$.

ANSWERS to the PRACTICE for TEST 4

50. True.

51. True. It factors as $y = (x-3)(x-7)$.

52. True.

53. False. They are at $(-1, 0)$ and $(7, 0)$.

54. False. This graph has no x-intercepts; the graph of $y = x^2 + 3x + 2$ has those two x-intercepts.

55. True.

56. False. The x-intercepts of $x^2 + 11x + 8$ are both negative, while the x-intercepts of $-x^2 + 11x - 8$ are both positive, the the additive inverses of the first two.

57. True.

58. True.

59. True. It is easy to see since each is the reflection of the other across the x-axis and two graphs reflected across the x-axis must have the same x-intercepts since points on the x-axis remain unchanged under a reflection across the x-axis.

60. True. One is just a translation of the other. That is clear when both are expressed in standard form.

61. True.

62. False. The largest possible product is 9, which occurs when the numbers are 1.5 and 6.

63. True.

64. False. The largest possible area is 900 square feet. There are four sides.

65. False. Before the 24 is added the answer must be at least -9; after the 24 is added, the smallest possible result is 15.

66. False. The maximum height is 5 meters above the ground. Use the vertex formula $c - \frac{b^2}{4a} = 0 - \frac{(-10)^2}{4(-5)} = 100/20 = 5$.

67. True.

68. False. They come out to be $\dfrac{x+1}{4}$ and $\dfrac{x-1}{4}$.

69. True.

70. True.

    It is a curiosity that the composites are not the same function because their domains are not the same. The formulas are the same, but the domains are different.

71. False. The maximum power is 250: $k = 0 - \dfrac{100^2}{4(-10)} = \dfrac{10000}{40} = 250$.