# Practice for Test 5

Math 130 *Kovitz* Fall 2018

The test is on Tuesday, November 27.

*Problems 1 through 65: True or false.*

1. The inverse function of $f(x) = \dfrac{3x+1}{5x-2}$ is $g(x) = \dfrac{2x+1}{5x-3}$.

2. The inverse of the function $f(x) = \dfrac{(x-7)^3 - 5}{2}$ is the function $g(x) = 2\sqrt[3]{x+7} + 5$.

3. The inverse of the function $f(x) = \dfrac{(x-7)^3 - 5}{2}$ is the function $g(x) = 2\sqrt[3]{x+7} + 10$.

4. The functions $f(x) = \dfrac{2x-7}{3}$ and the function $g(x) = \dfrac{3x+7}{2}$ are inverses of each other.

5. The functions $f(x) = \dfrac{2x-5}{3}$ and the function $g(x) = \dfrac{3}{2x-5}$ are inverses of each other.

6. The inverse of the function $y = x^3 - 1$ is the function $y = \sqrt[3]{x+1}$.

7. The function $y = |x|$ does not have an inverse function.

8. The inverse function of $f(x) = \dfrac{1}{x+1}$ is $g(x) = \dfrac{1-x}{x}$.

9. A function that is symmetric across the line $y = x$ must pass the horizontal line test and must be its own inverse.

10. A one-to-one function that is symmetric across the line $y = x$ is its own inverse.

11. The graph of $y = 2^x$ passes the Horizontal Line Test, lies in the first and second quadrants, has a $y$-intercept at $(0, 1)$, and has an asymptote on the negative $x$-axis.

12. On the graph of $y = 2^x$, if the 2nd coordinate of point B is the square of the first coordinate of point A, the first coordinate of point B must be twice the first coordinate of point A.

13. On the graph of $y = 2^x$, if the 1st coordinate of point B is 5 more than the 1st coordinate of point A, the 2nd coordinate of point B must be 5 times the 2nd coordinate of point A.

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14. The function $y = (1/2)^x$ is decreasing.

15. The function $y = \log_2 x$ is a decreasing function whose graph lies in the first and second quadrants and has an $x$-intercept of $(1, 0)$.

16. The graph of $y = \log_2 x$ has its asymptote on the negative $y$-axis.

17. The equation $y = \log_2 x$ defines a one-to-one function.

18. The function $y = \log_2 x$ has domain all positive reals and range all reals.

19. The value of $\log_3 41$ is not between 27 and 81, but it is between $\log_3 27$ and $\log_3 81$.

20. The value of $\log_2(1/17)$ is between $1/32$ and $1/16$.

21. The value of $\log_2(1/17)$ is between $-5$ and $-4$.

22. The value of $\log_2(-5)$ is between $-3$ and $-2$.

23. The value of $\log_8(5/6)$ is between $-1$ and $0$.

24. Since the functions $f(x) = \log_a x$ and $g(x) = a^x$ (with $a > 0$ and $a \neq 1$) are inverses of each other, it is surely true that $\log_a(a^p) = a^{\log_a p}$, assuming that $p$ is positive.

25. Since the functions $f(x) = \log_4 x$ and $g(x) = 4^x$ are inverses of each other, $4^{\log_4 \frac{1}{64}} = -3$.

26. Since the functions $f(x) = \log_3 x$ and $g(x) = 3^x$ are inverses of each other, $\log_3(3^7) = 81$.

27. Since the functions $f(x) = \log_3 x$ and $g(x) = 3^x$ are inverses of each other, $\log_3(3^0) = 1$.

28. When it is rewritten in exponential form, $\log b = c$ becomes $10^b = c$.

29. The inverse of the function $y = 10^x$ is the common logarithm function with equation $y = \log_{10} x$.

30. When it is rewritten in exponential form, $\log_9 2187 = 3.5$ becomes $9^{3.5} = 2187$.

31. In exponential form, $\log_2(32\sqrt{2}) = 5.5$ becomes $2^{5.5} = 32\sqrt{2}$.

32. In logarithm form, $4^{3/4} = 2\sqrt{2}$ becomes $\log_4\left(\frac{3}{4}\right) = 2\sqrt{2}$.

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33. The difference of the logarithms of two positive numbers is equal to the logarithm of the quotient of the two numbers.

34. The logarithm of the fifth root of a positive real number is equal to 1/5 of the logarithm of the number.

35. The logarithm of the square of a positive real number is equal to the square of the logarithm of the number.

36. $\log\left(\frac{1}{ab}\right) = -(\log a + \log b)$.

37. $\log 2.4 = \log 2 \times \log 0.4$.

38. $\log 2.4 = \log 2 + \log 0.4$.

39. $\log_2\left(\frac{1/8}{16}\right) = \log_2(1/8) - \log_2 16 = -3 - 4 = -7$.

40. $\log_4\left(13^{1/4}\right) = -1 \times \log_4 13$.

41. When $\log_b 2 = 1.637$ and $\log_b 2.72 = 2.363$, $\log_b 5.44 = 4$.

42. If $\log_b 2 = 1.637$ and $\log_b 2.72 = 2.363$, $\log_b 8 = 8 \times 1.637 = 13.096$.

43. When $\log_b 2 = 1.637$ and $\log_b 2.72 = 2.363$, $\log_b 8 = 3 \times 1.637 = 4.911$.

44. The only solution to the equation $\log_3(1 - 2x) + \log_3 x = -2$ is $x = 1/3$.

45. The only solution to the equation $\log_2(1-x) + \log_2 x = -2$ is $x = 1/2$.

46. The equation $\log_6(13 - x) + \log_6(x) = 2$ has two solutions.

47. The equation $\log_6(17 - 2x) + \log_6(x) = 2$ has one solution: $x = 4$.

48. The equation $\log_2(x + 3) + \log_2(-x) = 1$ has no real-numbered solutions.

49. The equation $\log_4(x + 1) + \log_4 x = 1/2$ has two real-numbered solutions, one of which is $x = 1$.

50. The equation $\log_2(x+1) + \log_2 x = 1$ has no real-numbered solutions.

51. The equation $\log_2(x-1) - \log_2 x = 1$ has no real-numbered solutions.

52. The equation $\log_3 x - \log_3(x - 1) = 2$ has only one real-numbered solution: $x = 1.125$.

53. The equation $\log(2x) - \log(x-2) = 1$ doesn't have an integer solution.

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54. The equation $\log(x) - \log(x-99) = 1$ has only one real-numbered solution, $x = 100$.

55. The equation $\dfrac{\log_2(5x-1)}{\log_2 2x} = 1$ has only one solution $x = 1/3$, while the equation $\log_2(5x-1) - \log_2 2x = 1$ has no solutions.

56. Change of base proves that $\log_{\sqrt{a}} b = \dfrac{1}{2}\log_a b$ in all cases.

57. It is always true that $\dfrac{1}{\log_b a} = \log_a b$, and it can be proved by changing the base of $\log_a b$ to base $b$.

58. $\log_b(-a) = -\log_b a$ in all cases.

59. Based on the approximation $\log 2 = 0.3$, $\log \sqrt[3]{2}$ is approximately equal to $\log \sqrt[10]{10}$.

60. Based on the approximation $\log 2 = 0.301$, an approximation for $\log_{64} 5$ is about $699/1806$, which makes some sense since $\log_{64} 5$ was estimated to be between $1/3$ and $1/2$.

    Hint: Start with $\log_{64} 5$ and change the base to base 10. Then rewrite 64 as a power of 2 and further simplify to get an approximate fraction.

61. Using change of base, the value of $\log_8 \sqrt[4]{2}$ comes out to be the exact fraction $1/12$.

62. Using change of base, $\log_{32} 2^{0.32}$ is shown to be exactly equal to 0.01.

63. The number of bacteria in a culture is increasing according to the law of exponential growth, the initial population is 200 bacteria, and the population after 8 hours will be double the population after 4 hours; so after 2 hours there will be exactly 300 bacteria.

64. The number of fruit flies in a colony is increasing according to the law of exponential growth, with the population after 2 hours exactly equal to 2 fruit flies and the population after 32 hours equal to 32 fruit flies; so the number of fruit flies after 8 hours will be exactly 8 fruit flies.

65. The number of fruit flies in a colony is increasing according to the law of exponential growth, with the population after 2 hours exactly equal to 2 fruit flies and the population after 32 hours equal to 32 fruit flies; so the number of fruit flies after $9\frac{1}{2}$ hours will be exactly 4 fruit flies.

Answers follow on subsequent pages.

**ANSWERS to the PRACTICE for TEST 2**

# Answers.

1. True.
2. False. The inverse is $\sqrt[3]{2x+5}+7$.
3. False. The inverse is $\sqrt[3]{2x+5}+7$.
4. True.
5. False. These two functions are reciprocals of each other.
   The inverse of $f$ is the function $f^{-1}(x) = \dfrac{3x+5}{2}$.
6. True.
7. True. It is not one-to-one; it fails the Horizontal Line Test.
8. True. Note that the function $g(x)$ has an alternate form $\dfrac{1}{x} - 1$.
9. True.
10. True.
11. True.
12. True.
13. False. It is 32 times as much.
14. True.
15. False. It is an increasing function which lies in the first and fourth quadrants.
16. True.
17. True.
18. True.
19. True.
20. False. It is between $\log_2(1/32)$ and $\log_2(1/16)$.
21. True.
22. False. The logarithms of negative numbers are not defined.
23. True.

**ANSWERS to the PRACTICE for TEST 2**

24. True.

25. False. It is equal to $\frac{1}{64}$.

26. False. It is equal to 7.

27. False. It is equal to 0.

28. False. It becomes $10^c = b$.

29. True.

30. True.

31. True.

32. False. It becomes $\log_4(2\sqrt{2}) = \frac{3}{4}$.

33. True.

34. True.

35. False. It is equal to twice the logarithm of the number.

36. True.

37. False. The other way around.
    $\log(2 \times 0.4) = \log 2 + \log 0.4$.

38. False. $\log(2 \times 0.4) = \log 2 + \log 0.4$.
    $2 \times 0.4$ is 0.8 not 2.4.

39. True.

40. False. It equals $(1/4) \times \log_4 13$.

41. True.

42. False. The value 8 is 2 cubed, so the answer is triple the log of 2.

43. True.

44. False. It also has a solution $x = 1/6$.

45. True.

46. True. They are $x = 4$ and $x = 9$.

47. False. It also has the solution $x = 4.5$.

**ANSWERS to the PRACTICE for TEST 2**

48. False. It has the solutions $x = -2$ and $x = -1$.

49. False. The only real-numbered solution is $x = 1$.

50. False. The unique solution is $x = 1$.

51. True.

52. True.

53. True. The unique solution is $x = 2.5$, but 2.5 is not an integer.

54. False. The one solution is $x = 110$. If the expression on the left were set equal to 2 instead of 1, 100 would be the correct solution.

55. False. The given solution is correct for the first equation but the second equation has the solution $x = 1$.

56. False. It proves that $\log_{\sqrt{a}} b = 2 \log_a b$.

57. True.

58. False. The logarithms of $a$ and $-a$ cannot both be defined as only one of the numbers $a$ and $-a$ could ever be positive.

59. True. One third of 0.3 is equal to one tenth of 1.

60. True. Note: Just after the change-of-base step, it was helpful—in the process of finding $\log 5$—to replace 5 by $10/2$ and take the log of that fraction.

61. True.

62. False. It is shown to be exactly equal to 0.064.

63. False. The base is $\sqrt[4]{2}$. So after two hours there will be
$$200(\sqrt[4]{2})^2 = 200\sqrt{2} \approx 282.8247 \text{ bacteria.}$$

64. False. The doubling time is $7\frac{1}{2}$ hours, so once 15 hours (two doubling times) have elapsed since the time of "after 2 hours," there will be 8 fruit flies. That will happen at time after 17 hours.

65. True. The doubling time is $7\frac{1}{2}$ hours, so after $9\frac{1}{2}$ hours the number of fruit flies would have doubled once since "after 2 hours." The 2 fruit flies would have become 4 fruit flies. Seven 7 hours and thirty minutes would have elapsed since "after 2 hours," and that is exactly one doubling time.