

Homework 15

(due December 1)

Math 130 *Kovitz* 2016

1. Explain the following apparent error in the sin function.

The equation

$$\sin\left(\frac{\pi}{6}\right) \approx \sin\left(\frac{3.141592654}{6}\right) \approx \sin .523598776 \approx .5$$

is true.

But $(.523598776, .5)$ is not a point on the unit circle since

$$(.523598776)^2 + (.5)^2 \approx .274155678 + .25 = .524155678 \neq 1,$$

meaning that

$$u^2 + v^2 \neq 1$$

and that the point $(.523598776, .5)$ is not on the unit circle.

The above statement is also true. In fact the only first-quadrant point on the unit circle for which $v = .5$ is $(.866025404, .5)$ (approximately).

How could $\sin(.523598776) \approx .5$ when the point $(.523598776, .5)$ is not even approximately on the unit circle. Explain. (Hint: ask yourself — of what is the unit circle a graph?)

2. (a) Graph $\sin x$ and $\cos x$, for x between 0 and 2π , on the same axes. Where do they intersect? Label the two points.
- (b) Using the unit circle $(u, v) = (\cos s, \sin s)$, find the points where $\sin s = \cos s$, for $0 \leq s \leq 2\pi$. Label them.
- (c) Show that the answers in parts (a) and (b) are exactly the same. Explain briefly.

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3. By using a graph of the unit circle draw the points that have the property that $\cos s = \frac{1}{2}$.

- (a) By looking at the graph, roughly estimate s for each such point. (Is there more than one answer to this question for each point?)
 (b) By looking at the graph, roughly estimate the value of $\sin s$ for each such point.

- *(c) It is surely true that $\sin s$ is a function—that means that $\sin s$ cannot have more than one value for a given s .

Explain the apparent contradiction to that fact by the two points which have the property that $\cos s = \frac{1}{2}$ but have different sines.

Also explain the apparent contradiction to $\cos s$ being a function by the fact that there are two values of s which make $\cos s = \frac{1}{2}$ true.

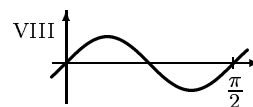
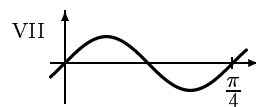
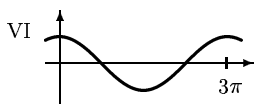
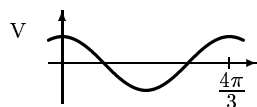
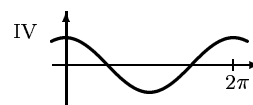
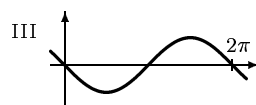
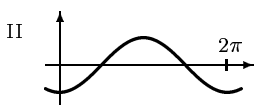
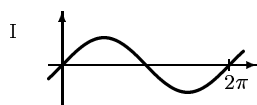
- (d) By using a graph of $\cos x$, locate some points which have the property that $\cos x = \frac{1}{2}$. By looking at the graph, roughly estimate x for at least four of those points.

4. (a) Find the period

- i. $\cos(7x)$
 ii. $\sin(\frac{x}{4})$
 iii. $\cos(2\pi x)$
 iv. $\cos(\frac{\pi x}{6})$

- (b) Match to the correct graph

- i. $\cos(\frac{2x}{3})$
 ii. $\sin(4x)$



5. By translation, graph $y = \sin(x - \frac{\pi}{6})$ for $0 \leq x \leq 2\pi$. Label with coordinates all intercepts and the peak and the valley.