Homework 10b

(due March 20, along with Challenge Problem 4)
Math 130 *Kovitz* 2018

In Problems 1–5, let f be the function given by

$$f(x) = \sqrt{x+4} - 5.$$

1. Decide if f is one-to-one. One method is to graph it and check the horizontal line test.

   Determine the domain and range of f.

   Determine the domain and the range of f^{-1}.

   Find the graph of the inverse of f by three methods:

   (a) by visually reflecting the graph of f across the diagonal line $y = x$,

   (b) by reflecting the endpoint and the intercepts of graph of f, connecting them with a smooth curve, and extending the curve to give it the same shape as the graph of f.

   (c) by producing an equation for the inverse function and graphing it over the known domain of f^{-1}. The formula over its entire implied domain and the inverse function we seek here are *not* the same, because the inverse has a smaller domain than the maximal domain of the underlying formula.

2. Find a formula for $f^{-1}(x)$ and state the domain.

3. Find $f^{-1}(f(-3))$ and $f(f^{-1}(-3))$.

4. Test the graph of f for symmetry across the line $y = x$.

*5. Compare $f^{-1}(-3)$ and $[f(-3)]^{-1}$. Are they equal?

In Problems 6–9, let f be the function given by

$$f(x) = \frac{6-x}{1-x}.$$

6. Write an equation of the inverse relation.

7. Test the graph of $y = \dfrac{6-x}{1-x}$ for symmetry across the line $y = x$

   (that is, determine whether $y = \dfrac{6-x}{1-x}$ is its own inverse).

8. Is the inverse relation a function?

9. If you answered yes to Problem 9, find a formula for $f^{-1}(x)$, and recheck your solution to Problem 8 in light of that formula.

10. Let $f(x) = \dfrac{5x-3}{4x-7}$, assuming f is one-to-one. Find a formula for $f^{-1}(x)$.

    As a check, find: $f(1)$, $f(2)$, $f^{-1}(-2/3)$, and $f^{-1}(7)$. Do these four calculations support the correctness of your formula for f^{-1}? Why?