Homework 15

(due May 3)

Math 130 *Kovitz* 2018

1. Explain the following apparent error in the sin function.

   The equation

   $$\sin\left(\frac{\pi}{6}\right) \approx \sin\left(\frac{3.141592654}{6}\right) \approx \sin .523598776 \approx .5$$

   is true.

   But $(.523598776, .5)$ is not a point on the unit circle since

   $$(.523598776)^2 + (.5)^2 \approx .274155678 + .25 = .524155678 \neq 1,$$

   meaning that

   $$u^2 + v^2 \neq 1$$

   and that the point $(.523598776, .5)$ is not on the unit circle.

   The above statement is also true. In fact the only first-quadrant point on the unit circle for which $v = .5$ is $(.866025404, .5)$ (approximately).

   How could $\sin(.523598776) \approx .5$ *when the point* $(.523598776, .5)$ *is not even approximately on the unit circle.* Explain. (Hint: ask yourself — of what is the unit circle a graph?)

2. (a) Graph $\sin x$ and $\cos x$, for x between 0 and 2π, on the same axes. Where do they intersect? Label the two points.

   (b) Using the unit circle $(u, v) = (\cos s, \sin s)$, find the points where $\sin s = \cos s$, for $0 \leq s \leq 2\pi$. Label them.

   (c) Show that the answers in parts (a) and (b) are exactly the same. Explain briefly.

TURN OVER

3. By using a graph of the unit circle draw the points that have the property that $\cos s = \frac{1}{2}$.

 (a) By looking at the graph, roughly estimate s for each such point. (Is there more than one answer to this question for each point?)

 (b) By looking at the graph, roughly estimate the value of $\sin s$ for each such point.

 *(c) It is surely true that $\sin s$ is a function—that means that $\sin s$ cannot have more than one value for a given s.

 Explain the apparent contradiction to that fact by the two points which have the property that $\cos s = \frac{1}{2}$ but have different sines. Also explain the apparent contradiction to $\cos s$ being a function by the fact that there are two values of s which make $\cos s = \frac{1}{2}$ true.
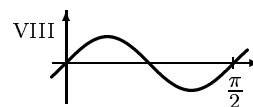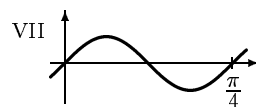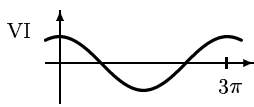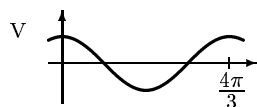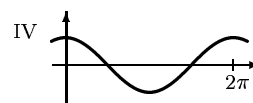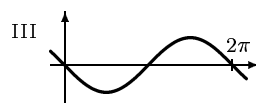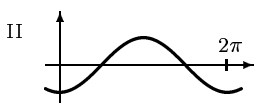
 (d) By using a graph of $\cos x$, locate some points which have the property that $\cos x = \frac{1}{2}$. By looking at the graph, roughly estimate x for at least four of those points.

4. (a) Find the period
 i. $\cos(7x)$
 ii. $\sin(\frac{x}{4})$
 iii. $\cos(2\pi x)$
 iv. $\cos(\frac{\pi x}{6})$

 (b) Match to the correct graph
 i. $\cos(\frac{2x}{3})$
 ii. $\sin(4x)$

5. By translation, graph $y = \sin(x - \frac{\pi}{6})$ for $0 \le x \le 2\pi$. Label with coordinates all intercepts and the peak and the valley.