

## Redo of Challenge Problem 4

(due April 10)

Math 130 *Kovitz* Spring 2018

Take an arbitrary point in the first quadrant of the unit circle and call it  $(a, b)$ . Let a triangle be drawn with vertices at  $(a, b)$  and at the left-most and right-most points of the unit circle.

Prove that this is a right triangle. Do not give numeric values to  $a$  and  $b$ ; just work with the letters and the fact that the point  $(a, b)$  is located on the unit circle. (What formula do  $a$  and  $b$  satisfy?)

Methods: choose one.

- Look at the slopes of the three sides of the triangle.
- Look at the lengths of the three sides of the triangle.
- Look at the area of the triangle in two different ways.

First let the base be the diameter. Then let the base be either of the two other sides.

- *A purely geometric argument:* Start by connecting the point  $(a, b)$  with the origin. Here do not use  $a$  and  $b$ —look at the relationships of the angles.
- *Alternate geometric method:* Rotate the upper unit semicircle  $180^\circ$  and then draw a line segment from the point  $(a, b)$  to its reflected point.