Challenge Problem 3

(due February 15)

Math 130 *Kovitz* Spring 2018

A point (a, b) is in the first quadrant of the unit circle.

Draw the reference right triangle and drop a perpendicular from the vertex of the right angle to the hypotenuse.

> The reference right triangle is constructed by drawing the radius to the point (a, b) and then dropping a perpendicular to the x-axis. The third side of the triangle is the line segment from the origin to the base of that perpendicular. The third side is a portion of the positive x-axis. To be clear, the right triangle has two of its vertices at the origin and at the point (a, b). The third vertex, that of the right angle, is on the positive x-axis.

- Find—in terms of a and b—the lengths of the two parts of the hypotenuse split by the perpendicular drawn.

  Which one is the length of the portion touching side a; and which is the length of the portion touching side b?

  Hint: Separately draw a careful picture for each triangle and then apply the rules for similar figures.

  The answer consists of two expressions, each in terms of a and/or b. No numbers yet.

- Now assume that leg a has length $3/5 = 0.6$. (Do work either in decimal or rational form.)

  For the original right triangle find the distance from the vertex of the right angle to the hypotenuse. This will equal the length of the perpendicular that was dropped.

  Hint: Draw the given right triangle to the numerical specifications and drop the perpendicular. Then, for clarity, draw separately the two smaller triangles that were formed. Are they similar to each other and similar to the orginal triangle?

  As a check, find b, the length of the other leg. Then use the formulas previously found to find the lengths of the two parts of that hypotenuse. They should add up to 1. Then show that all three right triangles are similar.

  Here the picture is being filled in with numerical values: no a or b will remain.