Challenge Problem 6

(due March 29)
Math 130 *Kovitz* Spring 2018

Even and Odd Functions

1. Let $f(x)$ be the function

$$f(x) = \frac{3^{2x} + 1}{3^x},$$

defined over the rational numbers.

What is the domain? the range? (Finding the range is challenging.)

In which quadrants must the graph of this function lie?

Decide whether it is odd, even, neither, or both.

To decide whether this function is odd or even, first look at $f(2)$ and $f(-2)$, $f(1)$ and $f(-1)$, $f(\frac{1}{2})$ and $f(-\frac{1}{2})$, simplifying each result to a single real number or decimal.

Is there a pattern that suggests odd or even?

Prove that conclusion using $f(-a)$ and $f(a)$ or $-f(a)$. Be sure that the relationship between $f(-a)$ and the other expression holds for all a in the domain of f.

2. Let $g(x)$ be the function

$$g(x) = \frac{3^{2x} - 1}{3^x},$$

defined over the rational numbers.

What is the domain? the range? (Finding the range is challenging.)

In which quadrants will the graph of this function lie?

Guess whether it is odd, even, neither, or both.

Find $g(2)$ and $g(-2)$, $g(1)$ and $g(-1)$, $g(\frac{1}{2})$ and $g(-\frac{1}{2})$, simplifying each result to a single real number or decimal.

How do these results support your previous guess?

Prove this function to be odd, even, or neither. Use $g(-a)$ and $g(a)$ or $-g(a)$, and show that the relationship holds for all a in the domain of g.