Comment on "Contextual Values of Observables in Quantum Measurements"

The Letter [1] of Dressel, Agarwal, and Jordan (henceforth called DAJ) introduces the concept of "contextual values" (CV) and claims that they lead to "a natural definition of a general conditioned average that converges uniquely to the quantum weak value in the minimal disturbance limit". They further claim that the usual notion of quantum weak values "can be subsumed as a special case in the CV formalism". The present Comment points out important gaps both in their mathematics and the reasoning which attempts to relate contextual values with weak values.

They never precisely define their "minimal disturbance limit", but if their "contextual values" in their "minimal disturbance limit" is taken to correspond to the usual notion of quantum "weak values" in the usual theory of quantum weak measurements, then it is not true that their "general conditioned average ... converges uniquely to the quantum weak value in the minimal disturbance limit", where they define"quantum weak value" A_w in their equation (7):

$$A_w = \frac{\mathrm{Tr}\,[\hat{\mathbf{E}}_f^{(2)}\{\hat{\boldsymbol{A}},\hat{\boldsymbol{\rho}}\}]}{2\mathrm{Tr}\,[\hat{\mathbf{E}}_f^{(2)}\hat{\boldsymbol{\rho}}]} \quad . \tag{7}$$

I shall refer to this as the "traditional weak value", since quantum weak values are not unique [2,3].

A major gap in their analysis occurs in the passage from their definition (6) of "the conditioned average of an observable" to the traditional weak value (7). Introducing a small "measurement strength" parameter g, they write the measurement operators as $\hat{\boldsymbol{M}}_{\boldsymbol{j}}(g) = \hat{\boldsymbol{U}}_{\boldsymbol{j}}(g)\hat{\boldsymbol{E}}_{\boldsymbol{j}}^{1/2}(g)$. This polar decomposition is surely possible, but then they go on to attempt to apply Stone's theorem to write $\hat{\boldsymbol{U}}_{\boldsymbol{j}}(g) = \exp[ig\hat{\boldsymbol{G}}_{\boldsymbol{j}}]$. But Stone's theorem requires that $g \to \hat{\boldsymbol{U}}_{\boldsymbol{j}}(g)$ be a one-parameter unitary *group*, i.e., that $\hat{\boldsymbol{U}}_{\boldsymbol{j}}(g_1+g_2) = \hat{\boldsymbol{U}}_{\boldsymbol{j}}(g_1)\hat{\boldsymbol{U}}_{\boldsymbol{j}}(g_2)$, which is surely not true in any generality in DAJ's context. For example, if it happened to be true for some particular $\hat{\boldsymbol{U}}_{\boldsymbol{j}}(g)$, it could be made false by nonlinearly rescaling g. There may be possibly be ways to get around this, but this error and the vagueness of DAJ's passage from (6) to (7) suggest that prudent readers should have a valid proof in hand before accepting that (6) implies (7).

Another problem is the lack of a precise definition of their "minimal disturbance limit". One gets the impression that it might have something to do with their condition $[\hat{\boldsymbol{G}}_{\boldsymbol{j}}, \hat{\boldsymbol{\rho}}] = 0$, but the physical motivation for this condition seems unclear.

If their "minimal disturbance limit" is supposed to correspond to what is usually called "weak measurement", then there is a natural definition of this. After a measurement associated with measurement operators $\{\hat{\boldsymbol{M}}_{\boldsymbol{j}}\}$ is made on a quantum system in state $\hat{\boldsymbol{\rho}}$, the system will subsequently be in state

$$\frac{\hat{\boldsymbol{M}}_{\boldsymbol{j}}\,\hat{\boldsymbol{\rho}}\,\hat{\boldsymbol{M}}_{\boldsymbol{j}}^{\dagger}}{\mathrm{Tr}\,[\hat{\boldsymbol{M}}_{\boldsymbol{j}}\,\hat{\boldsymbol{\rho}}\,\hat{\boldsymbol{M}}_{\boldsymbol{j}}^{\dagger}]}$$

with probability $\mathrm{Tr}\,[\hat{\boldsymbol{M}}_{\boldsymbol{j}}\,\hat{\boldsymbol{\rho}}\,\hat{\boldsymbol{M}}_{\boldsymbol{j}}^{\dagger}]$. Hence a natural (almost unique) definition of "weak" or "minimal disturbance" limit would be

$$\lim_{g\to 0} \frac{\hat{\boldsymbol{M}}_{\boldsymbol{j}}(g)\,\hat{\boldsymbol{\rho}}\,\hat{\boldsymbol{M}}_{\boldsymbol{j}}^{\dagger}(g)}{\mathrm{Tr}\,[\hat{\boldsymbol{M}}_{\boldsymbol{j}}(g)\,\hat{\boldsymbol{\rho}}\,\hat{\boldsymbol{M}}_{\boldsymbol{j}}^{\dagger}(g)]} = \hat{\boldsymbol{\rho}} \quad \text{for all } j.$$

Under this definition, their claim that (6) implies (7) in this limit can be rigorously shown to be false in general. Space limitations preclude giving the proof here, but it is not difficult and can be found in full in [4].

I thank the authors of DAJ for private communications clarifying this paper and regret that I am still unconvinced concerning the passage from (6) to (7).

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[1] J. Dressel, S. Agarwal, and A. N. Jordan, Phys. Rev. Lett. **104** 240401 (2010)
[2] Jozsa, R., Phys Rev A **76** 044103 (2007)
[3] S. Parrott, www.arXiv.org/quant-ph/0909.0295
[4] S. Parrott, www.arXiv.org/quant-ph/1102.4407