Comment on "Contextual Values of Observables in Quantum Measurements"

The Letter [1] of Dressel, Agarwal, and Jordan (henceforth called DAJ) introduces the concept of "contextual values" (CV) and claims that it leads to "a natural definition of a general conditioned average that converges uniquely to the quantum weak value in the minimal disturbance limit". They never precisely define their "minimal disturbance limit", but they do seem to define"quantum weak value" A_w in their equation (7):

$$A_w = \frac{\text{Tr }[\hat{\mathbf{E}}_f^{(2)}\{\hat{\boldsymbol{A}}, \hat{\boldsymbol{\rho}}\}]}{2\text{Tr }[\hat{\mathbf{E}}_f^{(2)}\hat{\boldsymbol{\rho}}]} \quad . \tag{7}$$

For pure states $\hat{\boldsymbol{\rho}}$, this reduces to the real part of the weak value obtained by the seminal paper [2].

A major error occurs in the passage from their definition (6) of "the conditioned average of an observable" to the quantum weak value (7). Introducing a small "measurement strength" parameter g, they write the measurement operators as $\hat{\boldsymbol{M}}_{\boldsymbol{j}}(g) = \hat{\boldsymbol{U}}_{\boldsymbol{j}}(g)\hat{\boldsymbol{E}}_{\boldsymbol{j}}^{1/2}(g)$. This polar decomposition is surely possible, but they go on to attempt to apply Stone's theorem to write $\hat{\boldsymbol{U}}_{\boldsymbol{j}}(g) = \exp[ig\hat{\boldsymbol{G}}_{\boldsymbol{j}}]$. But Stone's theorem requires that $g \to \hat{\boldsymbol{U}}_{\boldsymbol{j}}(g)$ be a one-parameter unitary *group*, i.e., that $\hat{\boldsymbol{U}}_{\boldsymbol{j}}(g_1 + g_2) = \hat{\boldsymbol{U}}_{\boldsymbol{j}}(g_1)\hat{\boldsymbol{U}}_{\boldsymbol{j}}(g_2)$, which is surely not true in any generality in DAJ's context. For a simple example, note that if it happened to be true for some particular $\hat{\boldsymbol{U}}_{\boldsymbol{j}}(g)$, it could be made false by nonlinearly rescaling g.

If $\hat{\boldsymbol{U}}_{\boldsymbol{j}}(g) = \exp[ig\hat{\boldsymbol{G}}_{\boldsymbol{j}}]$ were added as an *assumption*, it would be an extremely strong assumption which would be expected to hold for nontrivial $\hat{\boldsymbol{U}}_{\boldsymbol{j}}(g)$ (i.e., $\hat{\boldsymbol{U}}_{\boldsymbol{j}}(g)$ not multiples of the identity operator) only in unusual special cases. *A priori*, the unitary parts $\hat{\boldsymbol{U}}_{\boldsymbol{j}}$ of the measurement operators can be completely arbitrary. There is no reason to assume that for fixed j, the different $\hat{\boldsymbol{U}}_{\boldsymbol{j}}(g)$ should even commute, as they must if $\hat{\boldsymbol{U}}_{\boldsymbol{j}}(g) = \exp[ig\hat{\boldsymbol{G}}_{\boldsymbol{j}}]$.

The assumption $\hat{\boldsymbol{U}}_{\boldsymbol{j}}(g) = \exp(ig\hat{\boldsymbol{G}}_{\boldsymbol{j}})$ is so strong that the essence of the hypothesis for DAJ's claim to have established (7) is probably that the measurement operators be positive (i.e., that $\hat{\boldsymbol{U}}_{\boldsymbol{j}}$ be trivial). However, this is still an interesting hypothesis. My attempts to prove (7) or find a counterexample under the assumption that the measurement operators are positive have been unsuccessful, nor have I been able to obtain a correct proof from the authors. More detailed discussion of this and other aspects of DAJ can be found in [3].

Those thinking of building on the work of DAJ or citing it should be aware that the status of (7) may be uncertain. The authors would do a service to other workers in the field by removing this uncertainty, either by a retraction or making public a precise definition of their "minimal uncertainty limit" and a peer-reviewable proof that (7) does follow in this minimal uncertainty limit.

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[1] J. Dressel, S. Agarwal, and A. N. Jordan, Phys. Rev. Lett. **104** 240401 (2010)
[2] Aharonov, Y. , Albert, D. Z. , Vaidman, L. , Phys. Rev. Lett **60**, 1351-1354 (1988)
[3] S. Parrott, www.arXiv.org/quant-ph/1102.4407v3