COMMENT

Calculational error in the Corrigendum to Dressel and Jordan's "Sufficient conditions for uniqueness of the weak value"

2575 Bowers Rd. Gardnerville, NV 89410, USA
Department of Mathematics (retired), University of Massachusetts at Boston (for identification, not mail)
http://www.math.umb.edu/∼sp

E-mail: S_Parrotttoast2.net

PACS numbers: 03.65.Aa, 03.65.Ta

Abstract. A Corrigendum [3] to [1] presents a Lemma which attempts to fill a gap in the proof of the main result of the latter. This Comment observes that the proof of the Lemma is invalidated by a calculational error.

The article [1] by Dressel and Jordan presents a "General theorem" (GT) giving sufficient conditions for a previously introduced "general conditioned average" to evaluate to the traditional quantum weak value. This article will be abbreviated DJ below, along with its authors. The proof of the GT was questioned in [2], which pointed out a gap. Subsequently DJ published a Corrigendum [3] which presented a Lemma which was apparently supposed to fill the gap.

I dispute that the truth of the Lemma would fill the gap exposed in [2], thus proving the full GT. The Corrigendum does not address important issues raised in [2]

The Lemma's truth would only establish a special case of the GT. Though not mentioned in the Corrigendum, this special case was first presented in [2] as a Conjecture which was its focus. However, the point may be moot because the present Comment observes that the Corrigendum's attempted proof of the Lemma is invalidated by a calculational error.

For the reader's convenience, the Lemma's statement and the first few lines of its proof are reproduced:

"**Lemma.** *The singular values of the $M \times N$ dimensional matrix $F = P + g^n F_n$ with $M \leq N$ have maximum leading order of g^n, where $P = [p_1\vec{1} \dots p_n\vec{1}]$ and $F_n = [\vec{E}_1 \dots \vec{E}_N]$ such that $\sum_j p_j = 1$ and $\sum_j \vec{E}_j = \vec{0}$.*

Proof. .... Since $P^T F_n = 0$, the latter has the simple form $H = P^T P + g^{2n} F_n^T F_n, \dots$" [$H$ was previously defined as $H := F^T F$.]

The authors don't further explain their notation, but it is clear from the context that $\vec{1}$ stands for the column vector whose entries are all 1, and $[\vec{E}_1 \dots \vec{E}_N]$ for the matrix whose columns are the vectors $\vec{E}_j$. An example of such P and F_n is:

$$P := \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}, \quad F_n := \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix} \quad .$$

Then

$$P^T F_n = \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix} = F_n \neq 0.$$

The attempted proof of the Lemma does not actually require that $P^T F_n = 0$, but only its consequence that $P^T F_n + F_n^T P = 0$. However, that is false, too. Since the rest of the proof of the Lemma rests heavily on the latter claimed equality, (equivalent to $H = P^T P + g^{2n} F_n^T F_n$), it must be considered invalid as written.

Even if $P^T F_n + F_n^T P = 0$ were added as a hypothesis, I question whether the Lemma's proof is correct as written. However, I believe I know how to correct it, so I do regard the Lemma as established under this additional hypothesis. However, that weaker version of the Lemma would not be sufficient to obviously establish the Conjecture of [2], much less the GT.

References

[1] Dressel J and Jordan A N 2012 Sufficient conditions for uniqueness of the Weak Value, *J. Phys. A: Math. Theor.* **45** 015304 1-14, called DJ in the text.

[2] Parrott S Proof gap in "Sufficient conditions for uniqueness of the Weak Value by J. Dressel and A. N. Jordan, J. Phys. A **45** (2012) 015304, arXiv:1202.5604v6. Version 7 onward will contain more information concerning the Conjecture and the Lemma.

[3] Dressel J and Jordan A N Corrigendum: Sufficient conditions for uniqueness of the weak value, it J. Phys. A: *Math. Theor.* **46** (2013) 029501